

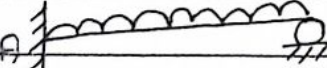
Structure $\rightarrow$ Analysis

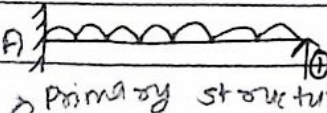
$\hookrightarrow$ design

Chapter 1: Introduction

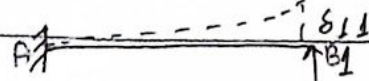
1. Types of structure based on material used
 - (a) Timber structure
 - (b) Steel structure
 - (c) R.C.C. (Reinforced Cement Concrete)
 - (d) Masonry structure (stone masonry & Brick masonry)
2. Structural Mechanics $\rightarrow$ A. DARGOV
 $\hookrightarrow$ The principles of mechanics used in analysing the structure is called Structural Mechanics
Principles $\rightarrow$ Eq^m
3. Two Basic Approaches of Structural Analysis (for statically indeterminate structures analysis)
 - (a) Force Method (Approach) / Flexibility Method / Compatibility Method
 - (b) Displacement Method (Approach) / Stiffness Method

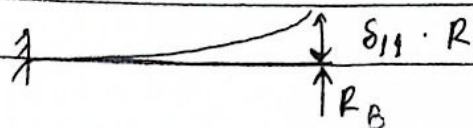
* Force Method

 (Propped cantilever) $\rightarrow$ indeterminate

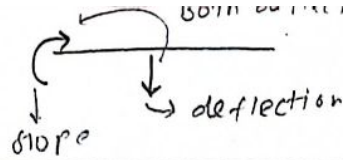
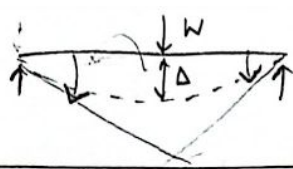
 (Primary structure (determinate)) $\rightarrow$ removing extra support

$\Delta_{11} = \frac{wL^4}{8EI}$ $\rightarrow$ determinate (Cantilever)



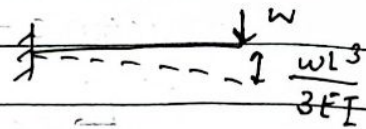


forces which are not absolutely essential for stability of str



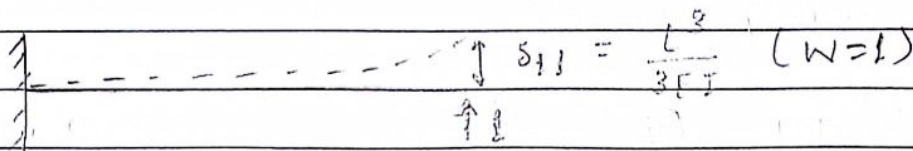
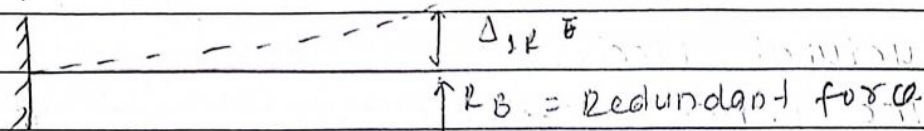
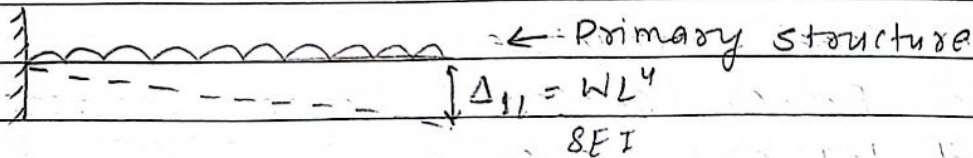
Compatibility condition

$$\Delta_{1L} + \Delta_{1R} = \Delta_f = 0$$



$$\text{or, } -\frac{WL^4}{8EI} + \frac{L^3 R}{3EI} = 0$$

Force method: (force unknown first then writing compatibility condition)



compatibility condition,

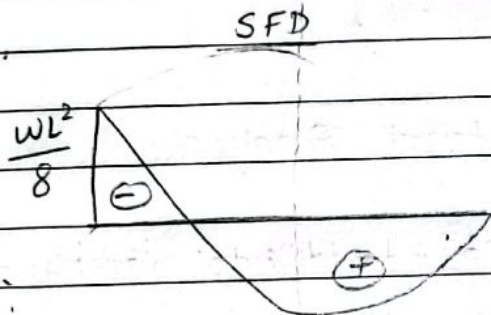
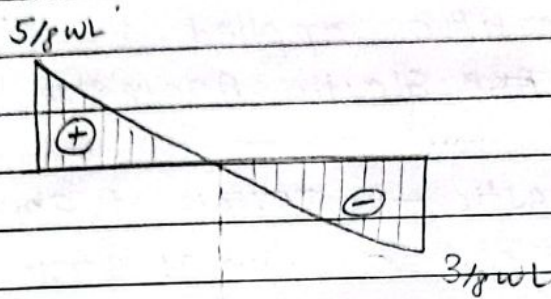
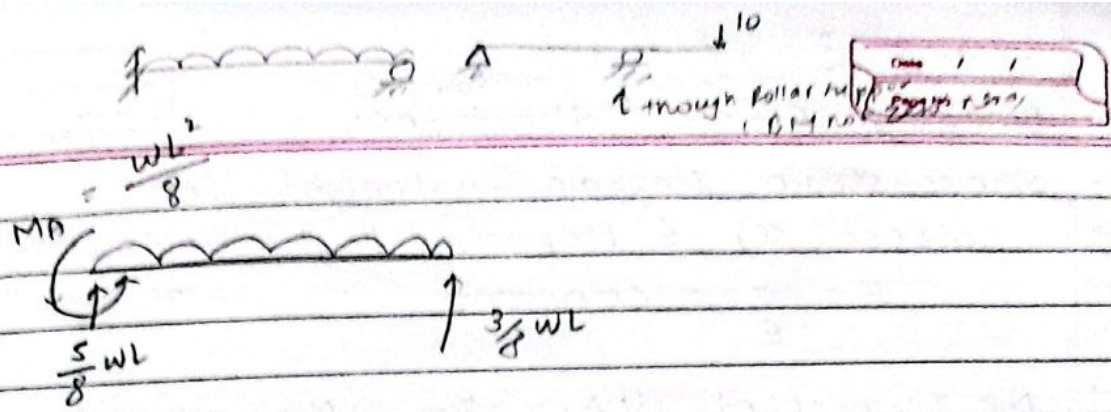
$$\Delta_B = 0$$

compatibility equation,

$$\Delta_{1L} + \Delta_{1R} = 0$$

$$-\frac{WL^4}{8EI} + R_B \frac{L^3}{3EI} = 0$$

$$\left[R_B = \frac{3}{8} WL \right]$$



$$M_x = \frac{5}{8}wLx - \frac{wL^2}{8} - \frac{wx^2}{2}$$

Force Method

It is a method for analysing statically indeterminate structure in which force quantities are considered as unknown and these are determined by solving the compatibility equations.

Displacement Method (for SDA software)

→ It is a method for analysing statically indeterminate structures in which displacements of joints are considered as unknown quantities and these are determined by writing equilibrium equations. (solving)

→ Then, the end moments of each member are determined by using slope deflection equations.

Linearly Elastic Structures:-

- Stress-strain Diagram → straight line
 - Stress (σ) $\leq$ Proportional limit value (σ_p)
 - $E = \frac{\sigma}{\epsilon} = \text{constant}$
 - No permanent deformation after removal of loading
 - Principle of superposition applied.
 - Analysis is called ~~Anal~~ Elastic Analysis.
 - Obeys Hooke's law
- For Linearly Inelastic → reverse of above

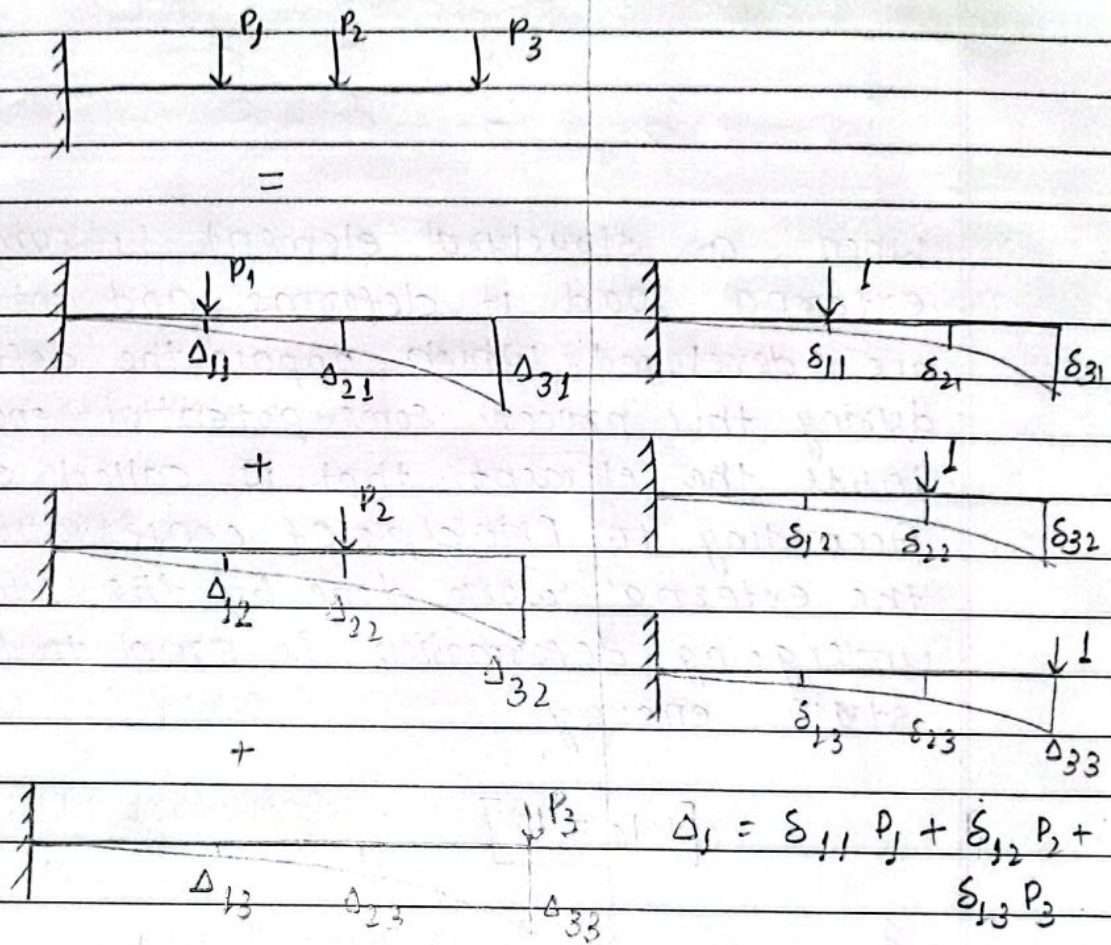
Non-linearity in Structural Analysis:

- Two types
 - (i) Material non-linearity → due to material → $\sigma > \sigma_p$
 - (ii) Geometrical non-linearity → due to excessive deformation.

Computer Based Analysis:

principle of superposition

linearly elastic
temp variation? can't apply



$$\Delta_1 = \Delta_{11} + \Delta_{12} + \Delta_{13}$$

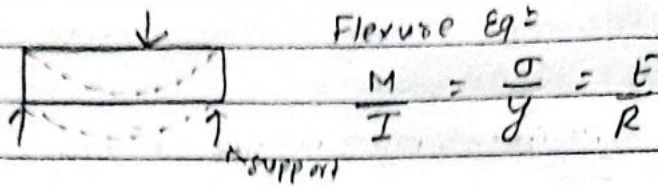
$$\Delta_2 = \Delta_{21} + \Delta_{22} + \Delta_{23}$$

$$\Delta_3 = \Delta_{31} + \Delta_{32} + \Delta_{33}$$

Chapter-2:

Strain Energy

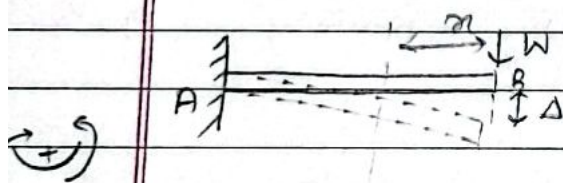
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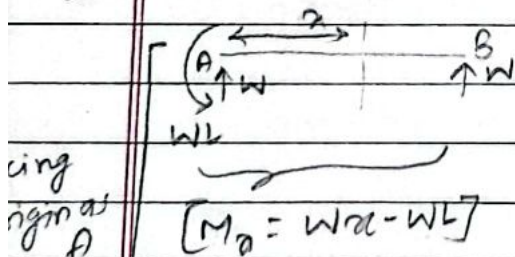
When a structural element is subjected to external load it deforms and internal stresses are developed which oppose the deformation, during this process some potential energy is stored inside the element that is called strain energy. According to Principle of conservation of Energy, the external work done by the load while undergoing deformation is equal to the internal strain energy.

$$[W_e = U]$$

Real-Work Method:



$$W_e = \frac{1}{2} W \Delta$$



$$U = \int \frac{M^2 dx}{2EI}$$

considering B as origin

$$= \int_0^L \frac{(-Wx)^2 dx}{2EI}$$

$$= \frac{W^2}{2EI} \times \frac{L^3}{3}$$

$$= \frac{W^2 L^3}{6EI}$$

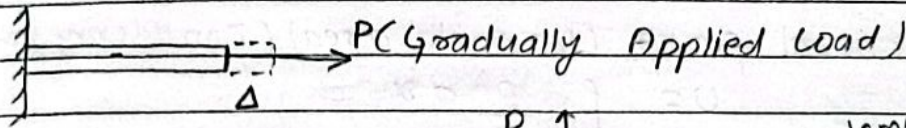
Equating,

$$W_e = U$$

$$\frac{1}{2} W \Delta = \frac{W^2 L^3}{6EI}$$

$$\left[\Delta = \frac{WL^3}{3EI} \right] \text{ deflection (using Real-Work Method)}$$

Strain Energy and Complementary Strain Energy



$$P \propto \Delta$$

$$P = K \Delta$$

For intermediate state,

$$P' = K \Delta'$$

Work done,

$$dU = P' \cdot d\Delta$$

Total Work done

$$U = \int_0^{\Delta} P' d\Delta$$

$$= \int_0^{\Delta} K \Delta' d\Delta' = \frac{1}{2} K \Delta^2$$

$$= \frac{1}{2} \left(\frac{P}{\Delta} \right) \Delta^2 = \frac{1}{2} P \Delta$$

$$\left[\text{Strain energy } (U) = \frac{1}{2} P \cdot \Delta = \text{Work Done} \right] \checkmark$$

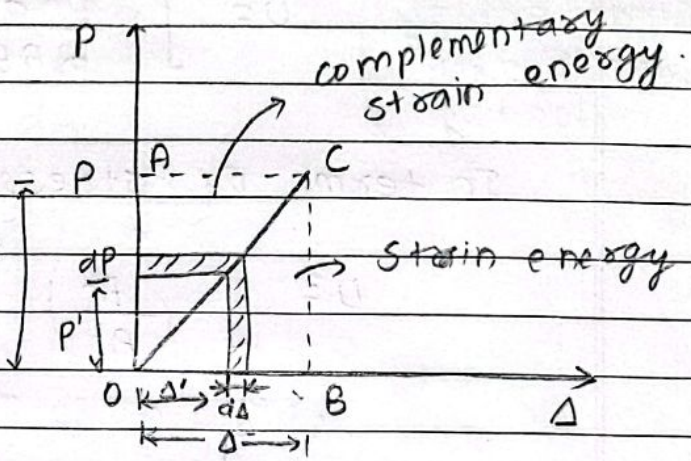
= Area of triangle OBC

(i.e. Area under the load

deformation curve represents

the strain energy and work done)

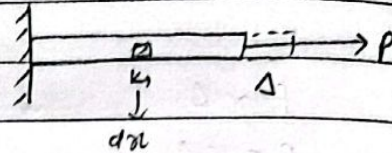
✓ Area above the load deformation curve represents complementary strain energy.



Also,

$$\text{Energy: } U = \frac{PA}{2} = \frac{P}{2} \left(\frac{PL}{AE} \right) = \frac{P^2 L}{2AE}$$

General form (variable area) (Considering small area)
 energy to axial loading $U = \int \frac{P^2 dx}{2AE}$



In terms of stress,

$$U = \int \left(\frac{P^2}{A^2} \right) \frac{A dx}{2E} = \frac{\sigma^2}{2E} \int dV = \int \frac{\sigma^2 dV}{2E}$$

($dV = \frac{\sigma^2 dV}{2E}$)

$\frac{dU}{dV}$ = Strain energy density
 (strain energy stored per unit volume)

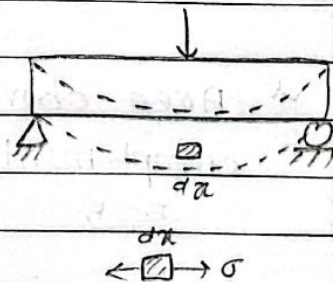
↳ (Modulus of Resilience)
 (constant for given material)
 ↳ gives capacity of structure to resist load.

General formula, (In terms of stress)

$$U = \int \frac{\sigma^2 dV}{2E} \rightarrow \text{used in bending, shear, axial.}$$

strain energy due to Bending (U_b)

$$\frac{M}{I} = \frac{\sigma}{y} \Rightarrow \sigma = \frac{M y}{I}$$



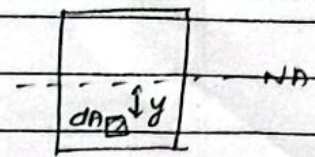
$$U_b = \int \frac{\sigma^2 dV}{2E}$$

$$\text{Thus, } U_b = \int \frac{\left(\frac{M}{I} y\right)^2 dA d\alpha}{2E}$$

$$= \int \frac{M^2 y^2 dA d\alpha}{2 I^2 E}$$

$$= \int \frac{M^2 I d\alpha}{2 I^2 E}$$

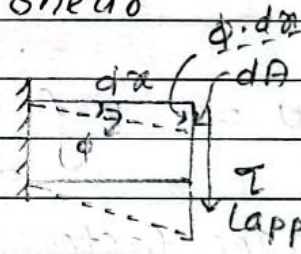
$$\left[U_b = \int \frac{M^2 d\alpha}{2EI} \right]$$



$$\int y^2 dA = I_{NA}$$

Strain energy due to shear

For elemental strip,



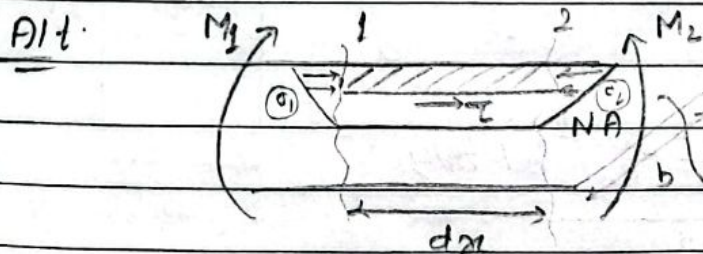
$$dU = \frac{1}{2} (dP) \cdot \Delta$$

$$U = \int \frac{1}{2} (\tau \cdot dA) \phi d\alpha$$

$$\left[\phi = \frac{\tau}{G} \right]$$

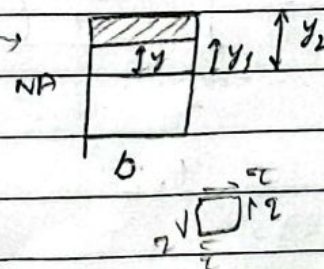
$$= \int \frac{1}{2} \tau dA \frac{\tau}{G} d\alpha \quad (\text{S.F. is a variable BM case})$$

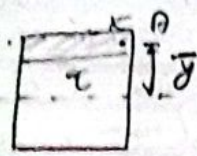
$$U = \int \frac{\tau^2 dV}{2G}$$



$$\sigma = \frac{M}{I} y$$

$$\int_{y_1}^{y_2} \sigma_1 dA - \int_{y_1}^{y_2} \sigma_2 dA = \tau d\alpha b$$





$$\tau = \frac{VS}{Ib} \quad \phi = S = A\bar{y} \\ [S = \phi]$$

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$$U = \int \frac{\tau^2 dV}{2G}$$

$$= \int \left(\frac{V^2 \phi^2}{I^2 b^2} \right) \frac{dA \cdot da}{2G} \quad \left[\tau = \frac{VS}{Ib} \right] \leftarrow \text{for section of beam.}$$

$I, b, \phi \rightarrow$ dimension (shape)

$V \rightarrow S.F$

$$= \int_A \frac{\phi^2 A dA}{I^2 b^2} \cdot \int \frac{V^2 da}{2GA}$$

$\int \frac{\phi^2 A dA}{I^2 b^2} \rightarrow$ depends upon shape
 K (shape factor)

$$\left[U = K \int \frac{V^2 da}{2GA} \right]$$

Shape factors (K):

$$K_{\square} = 1.2$$

$$K_{\Delta} = 4/3$$

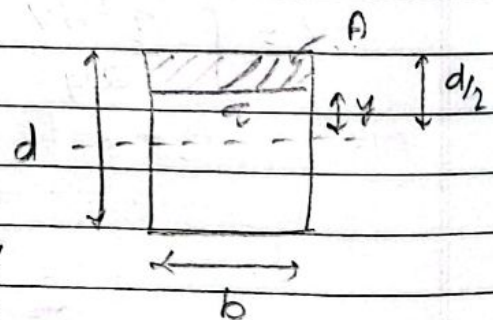
$$K_O = \frac{10}{9}$$

For rectangle

$$K = \int \frac{\phi^2 A dA}{I^2 b^2}$$

$$= \int_{-d/2}^{d/2} \frac{\frac{b^2}{4} \left(\frac{d^2}{4} - y^2 \right)^2 bd \cdot b dy}{I^2 b^2}$$

$$= 1.2$$



$$\phi = A\bar{y}$$

$$\phi = \left(\frac{d}{2} - y \right) b$$

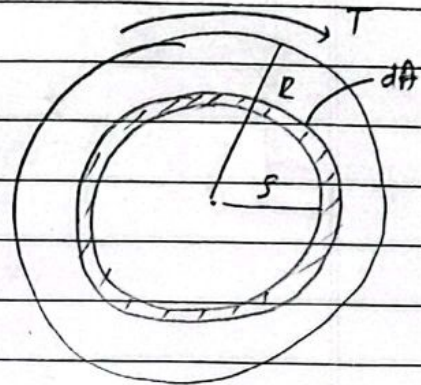
$$\left(y + \frac{1}{2} \left(\frac{d}{2} - y \right) \right)$$

$$\left(\phi = \frac{b}{2} \left(\frac{d^2}{4} - y^2 \right) \right)$$

Strain Energy due to Torsion: (Shear stress)

$$U = \int \frac{\tau^2 dV}{2G}$$

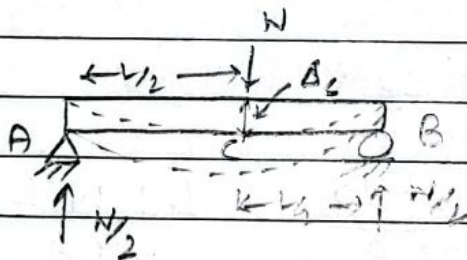
Torsional Equation: $\frac{\tau}{R} = \frac{\tau}{\rho} = \frac{T}{J} = \frac{G\phi}{L}$



$$U = \int \frac{T^2}{2GJ^2} \rho^2 dA \cdot dx$$

$$= \int \frac{T^2 J dx}{2GJ^2} \quad [J = \int \rho^2 dA]$$

$$[U = \int \frac{T^2 dx}{2GJ}]$$



$\Delta_c \Rightarrow$

UDL $\times$
Real-Work Method

Only used to find deflection where the load is applied if more than one load is applied $\Rightarrow$ difficult to apply

$$W_e = \frac{1}{2} W \cdot \Delta_c$$

$$U = 2 \int_0^{L/2} \left(\frac{W}{2} \cdot x \right)^2 \frac{dx}{2EI}$$

$$= \frac{W^2}{4EI} \left(\frac{x^3}{3} \right)_0^{L/2}$$

$$= \frac{W^2}{4EI} \times \frac{1}{3} \times \left(\frac{L^3}{8} \right)$$

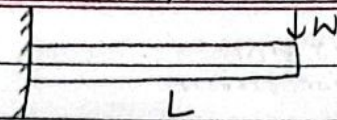
$$= \frac{W^2 L^3}{96EI}$$

$$W_e = U$$

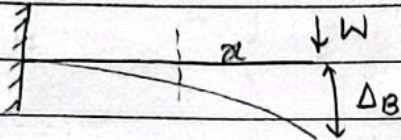
$$\text{or, } \frac{1}{2} W \Delta_c = \frac{W^2 L^3}{96EI}$$

$$\text{or, } \left[\Delta_c = \frac{W L^3}{48EI} \right]$$

Real Work Method



$$\Delta_B = \frac{WL^3}{3EI}$$



$$W_e = \frac{1}{2} W \cdot \Delta_B$$

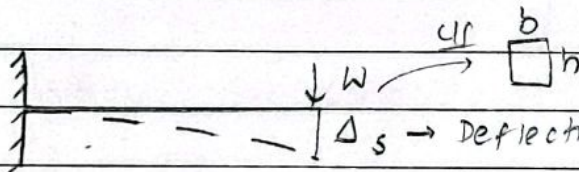
$$U = \int \frac{M^2 dx}{2EI}$$

$$= \int_0^L \frac{(-Wx)^2 dx}{2EI}$$

$$= \frac{W^2 L^3}{6EI}$$

$$W_e = U$$

$$\Rightarrow \text{Deflection due to Bending } (\Delta_B) = \frac{WL^3}{3EI}$$



$$W_e = \frac{1}{2} W \cdot \Delta_S$$

$$U = K \int_0^L \frac{v^2 dx}{2GA} = \frac{1.2 W^2 L}{2GA}$$

$$W_e = U$$

$$\Rightarrow \Delta_S = \frac{1.2 WL}{GA}$$

$$\text{Total deflection } (\Delta) = \Delta_B + \Delta_S$$

$$\Delta = \frac{WL^3}{3EI} + \frac{1.2 WL}{GA}$$

$$= \frac{WL^3}{3EI} \left(1 + \frac{3EI \times 1.2}{L^2 GA} \right)$$

$$= \frac{WL^3}{3EI} \left[1 + \frac{3E \left(\frac{bh^3}{12} \right) \times 1.2}{L^2 G \times b \times h} \right]$$

$$\Delta = \Delta_B \left[1 + \frac{0.3 h^2}{L^2} \frac{E}{G} \right]$$

For steel, $\frac{E}{G} = 2.5$

$$\text{Then, } \Delta = \Delta_B \left[1 + 0.75 \frac{h^2}{L^2} \right]$$

Generally, $L = 10h$ (Say)

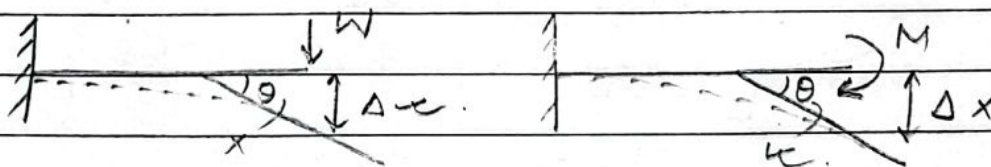
$$\Delta = \Delta_B [1 + 0.0075]$$

Thus, In normal beam deflection due to bending is more significant than deflection due to shear.

Limitations of Real-Work Method:

- (i) can be applied for single load and deflection at same point.
- (ii) Not applied for in case of UDL
- (iii) Not applied in case of deflection at other location than the position of load.

[For applied load $\rightarrow$ slope can't be found out
For applied moment $\rightarrow$ deflection can't be found out]



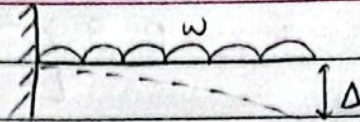
$$W_e = \frac{1}{2} M \theta$$

$$U = \int_0^L \frac{M^2 d\theta}{2EI} = \frac{M^2 L}{2EI}$$

$$W_e = U$$

$$\theta = \frac{ML}{EI}$$

For UDL



Strain energy can be found out (Wint)

But $W_{ext.}$ can't be found out

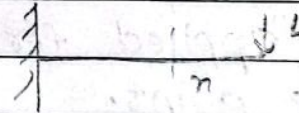
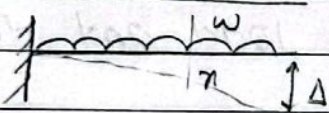
(However, virtual work done can be calculated)

$$U = \int \frac{M^2 dx}{2EI} = \int_0^L \left(\frac{-wx^2}{2} \right)^2 dx$$

$$= \frac{w^2}{8EI} \times \frac{L^5}{5}$$

$$= \frac{w^2 L^5}{40EI}$$

Virtual Work → for all types of loading



No $\frac{1}{2}$ as it is considered that 1 load is applied previously

$1 \times \Delta = \int \frac{M m dx}{EI}$

↑ due to real loading
↓ due to virtual loading

Virtual load
Deflection due to Real load

$$1 \times \Delta = \int_0^L \left(\frac{-wx^2}{2} \right) (-x) \frac{dx}{EI}$$

$$\left[\Delta = \frac{w L^4}{8EI} \right]$$

(Sudden load $\rightarrow$ Load (let from zero ht.)
 Gradual load $\rightarrow 0 \rightarrow$ Load
 Impact load $\rightarrow$ sudden load applied from certain ht.)

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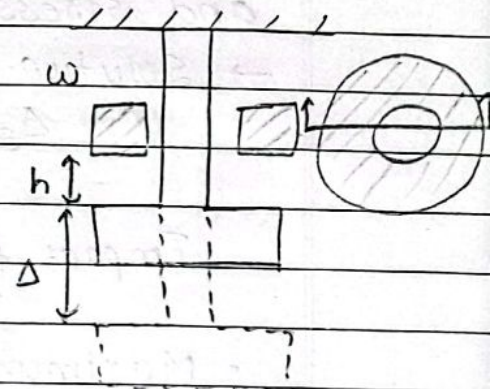
Impact Load

• Impact factor

Assuming no loss of energy due to heat, sound and vibration.

Potential energy = Internal work

$$W(h+\Delta) = \frac{\Delta^2 AE}{2L}$$



$$\text{or, } \frac{2WhL^2}{AE} + \frac{2W\Delta L^2}{AE} - \Delta^2 = 0$$

$$\text{or, } 2h \underbrace{\Delta_{\text{static}}}_{\frac{WL}{AE}} + 2\Delta\Delta_{\text{st}} - \Delta^2 = 0$$

$$\text{or, } \Delta^2 - 2\Delta_{\text{st}}\Delta - 2h\Delta_{\text{st}} = 0$$

Solving the quadratic.

$$\Delta = \frac{+2\Delta_{\text{st}} + \sqrt{4\Delta_{\text{st}}^2 + 8h\Delta_{\text{st}}}}{2} \quad (\text{taking +ve sign only})$$

$$\Delta = \Delta_{\text{st}} + \sqrt{\Delta_{\text{st}}^2 + 2h\Delta_{\text{st}}}$$

$$\left\{ \Delta = \Delta_{\text{st}} \left[1 + \sqrt{1 + \frac{2h}{\Delta_{\text{st}}}} \right] \right\} \text{ etc.}$$

Here, $\left(1 + \sqrt{1 + \frac{2h}{\Delta_{\text{st}}}} \right)$ is impact factor or dynamic multiplier.

• If, $h=0$ (i.e. sudden loading condition)

$\hookrightarrow$ Then, impact factor = 2

(Q) Find maximum instantaneous deflection and stress.

→ Solution.

$$\Delta_{st} = \frac{WL^3}{3EI}$$

$$\text{Impact factor (I.f.)} = 1 + \sqrt{1 + \frac{2h}{\Delta_{st}}}$$

• Maximum instantaneous deflection (Δ)

$$\Delta = \text{I.f.} \times \Delta_{st} = \left(1 + \sqrt{1 + \frac{2h}{\Delta_{st}}}\right) \left(\frac{WL^3}{3EI}\right)$$

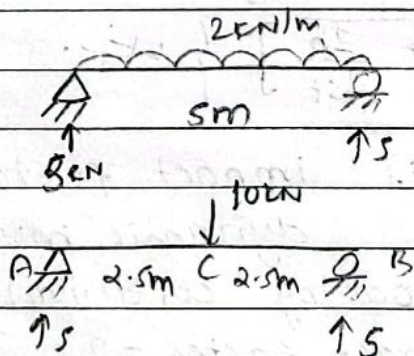
• For stress, (maximum stress → at max. moment)

$$\sigma_{st} = \frac{M}{I} y \quad [M_{\max} \text{ at support}]$$

$$(\sigma_{st})_{\max} = \frac{WL}{I} \times y \quad M_{\max} = WL$$

$$\text{Maximum instantaneous stress} = \text{I.f.} \times (\sigma_{st})_{\max}$$

Compare strain energy



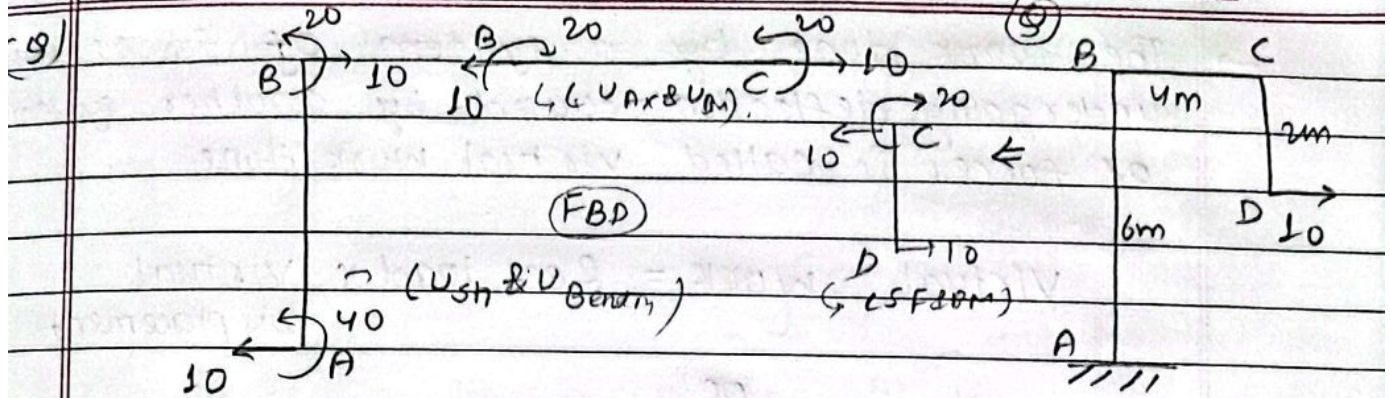
$$U = \int_0^5 \frac{(5x - \frac{2}{2}x^2)^2}{2EI} dx$$

$$U = 2 \int_0^{2.5} \frac{(5x)^2}{2EI} dx$$

(Break by 10 kN)

Symmetry on both side of C
so $\times 2$

$$U = U_{AC} + U_{CB}$$



$$U_s = 1.2 \int_0^2 \frac{10^2 dx}{2GA} + 1.2 \int_0^6 \frac{10^2 dx}{2GA}$$

$$U_A = \int_0^4 \frac{10^2 dx}{2AE}$$

$$U_B = \int_0^2 \frac{(10m)^2 dx}{2EI} + \int_0^4 \frac{20^2 dx}{2EI} + \int_0^6 \frac{(10m-40)^2 dx}{2EI}$$

(D at origin) (const BM) (A at origin)

$$\frac{1}{2} \times 10 \times \Delta_D = \text{Total strain energy } (U_s + U_A + U_B)$$

Horizontal

Deflection.

Chapter-3

Virtual Work Method

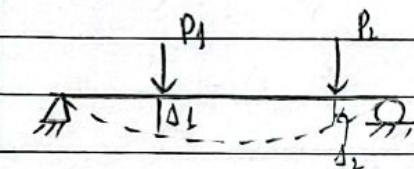
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→ The work done by a system of forces while undergoing deflection caused by another system of forces is called virtual work done.

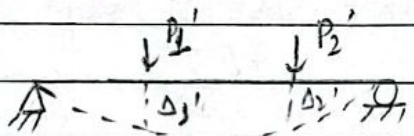
$$\text{Virtual Work} = \text{Real load} \times \text{Virtual Displacement}$$

OR

$$\text{Virtual Work} = \text{Virtual Load} \times \text{Real Displacement}$$



$$\sum P \Delta = P_1 \Delta_1 + P_2 \Delta_2$$



(Virtual load → considered applied already rather gradually so NO $\frac{1}{2}$ is done for calculating virtual work)

$$\sum P \Delta = P_1 \Delta_1 + P_2 \Delta_2$$

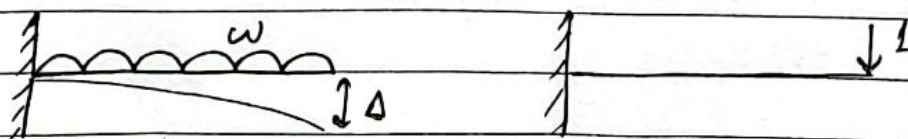
$$\sum P' \Delta = P_1' \Delta_1 + P_2' \Delta_2$$

(deflection & loading of diff. system)

$[\sum P \Delta = \sum P' \Delta] \rightarrow$ Bati's Theorem.

- Principle of Virtual Work for deformable bodies
In deformable bodies, the external virtual work done is equal to the internal virtual work done

(Virtual load → applied where displacement is to be found)



$$\text{External virtual work done} = 1 \times \Delta$$

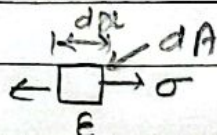
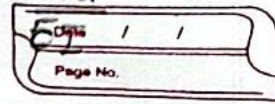


fig.

$$\int \frac{Mm dx}{EI} = \int \frac{M_p M_q dx}{EI} = \int MM^* dx$$



Internal virtual work done = $\int (\sigma^* dA) (\epsilon dx)$

$\uparrow$ virtual stress (due to virtual loading) $\uparrow$ real displacement
 $= \int \sigma^* \epsilon dv$ moment due to virtual load

$$\frac{\sigma^*}{y} = \frac{m}{I} \Rightarrow \sigma^* = \frac{m}{I} y$$

Internal virtual work done = $\int \left(\frac{m}{I} y \right) \left(\frac{\sigma}{E} \right) dA \cdot dx$

$= \int \left(\frac{m}{I} y \right) \left(\frac{M y}{I} \right) \frac{1}{E} dA \cdot dx$
 $= \int \frac{Mm dx}{EI^2} (y^2 dA)^I$
 $= \int \frac{Mm dx}{EI}$

Frame and Beam

$\delta \Delta = \sum \int \frac{Mm dx}{EI}$ → Product Integral

applied where real displacement to be found out.

$M \rightarrow$ Moment due to real load
 $m \rightarrow$ Moment due to virtual load

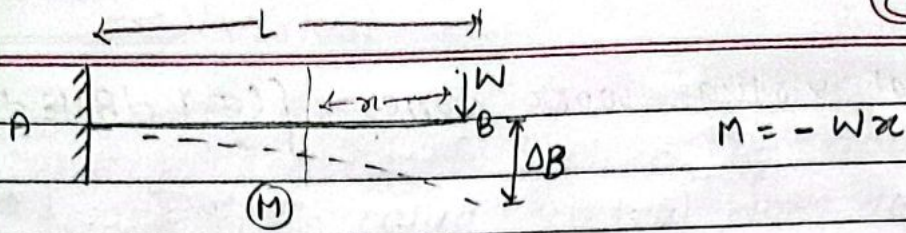
Graphically,

$$\sum \int \frac{Mm dx}{EI} = \frac{A_M \cdot y_c}{EI}$$

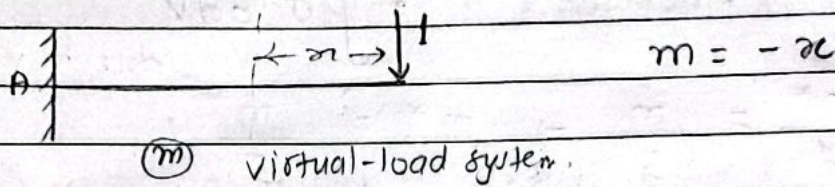
$A_M \rightarrow$ Area under M

$y_c \rightarrow$ ordinate of central pt. of m (centroid)

↳ Ordinate of m diagram corresponding to A_M diagram.



Real-load system



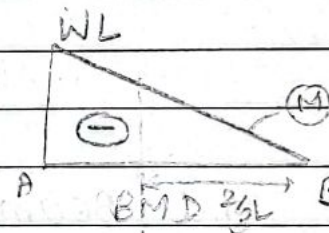
(m) virtual-load system.

we consider same origin

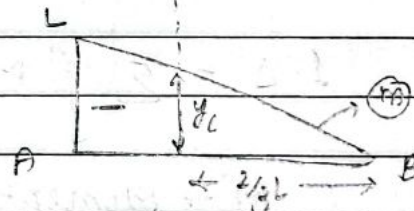
$$1 \cdot \Delta_B = \int_0^L \frac{(-Wx)(-x)}{EI} dx$$

$$\left[\Delta_B = \frac{WL^3}{3EI} \right] \text{ we.}$$

For Real-load system:



For virtual load system:



$$\frac{L}{L} = \frac{y_c}{2/3L}$$

$$\Rightarrow \left[y_c = \frac{2}{3}L \right]$$

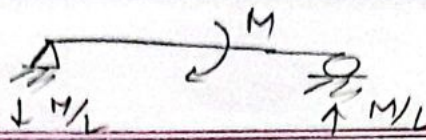
$$\Delta_B = \frac{A_M \times y_c}{EI} = \frac{\left(-\frac{1}{2} \times L \times WL \right) \times \left(\frac{2}{3}L \right)}{EI}$$

$$\left[\Delta_B = \frac{WL^3}{3EI} \right]$$

↳ Considered virtual load downward

& ans. is +ve so,

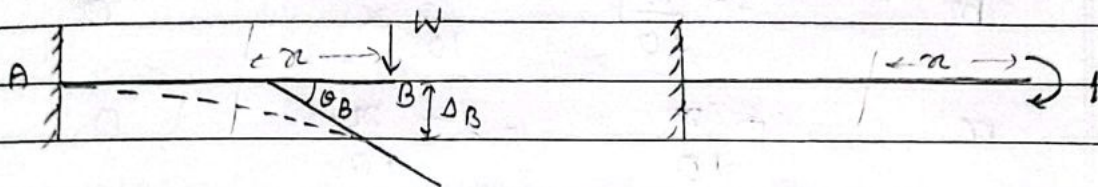
Δ_B is downward.



For deflection $\rightarrow$ unit load is applied as virtual load

For rotation $\rightarrow$ unit moment is applied as virtual load.

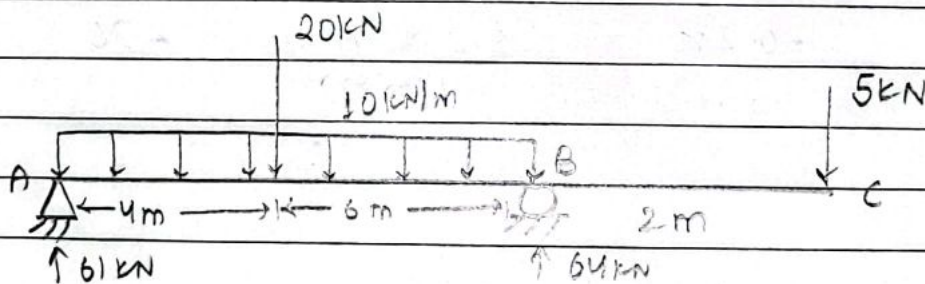
For rotation



$$1 \cdot \theta_B = \int_0^L \frac{(-Wx)(-1)}{EI} dx$$

$$\theta_B = \frac{WL^2}{2EI}$$

(D)



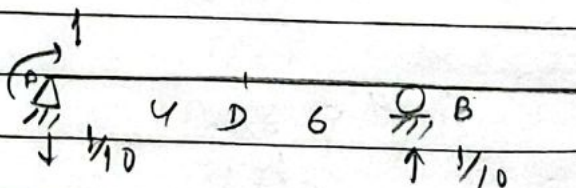
$$\theta_A = ? , \theta_B = ? , \Delta_D = ? , \Delta_C = ? , \theta_C = ?$$

$\rightarrow$ Solution,

Calculation of R_{xn} .

$$\left. \begin{aligned} [R_B = 64 \text{ kN}(\uparrow)] \\ [R_A = 61 \text{ kN}(\uparrow)] \end{aligned} \right\} \text{Real-system}$$

Virtual-load system for θ_A



Position	AD	DB	BC
Origin	A	B	C
Limit	0-4	0-6	0-2

$$M \quad +61x - 5x^2 \quad 64x - 5(x+2) - 5x^2 \quad -5x$$

$$59x - 5x^2 - 10$$

(a) $m \quad 1 - \frac{x}{10} \quad \frac{1}{10}x \quad 0$

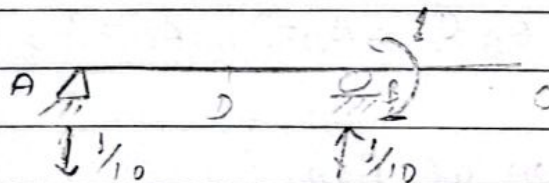
(b) $m \quad -\frac{x}{10} \quad -1 + \frac{x}{10} \quad 0$

(c) $m \quad \frac{x}{10} \quad 1 - \frac{x}{10} \quad 1$

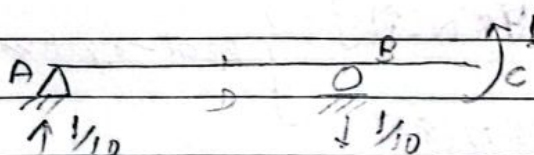
(d) $m \quad 0.6x \quad 0.4x \quad 0$

(e) $m \quad -0.2x \quad 1.2x \quad -x$
 $-1(x+2)$

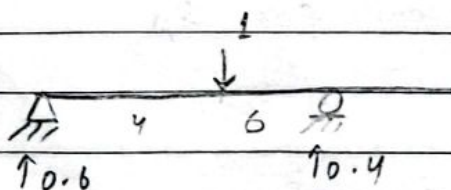
For θ_B



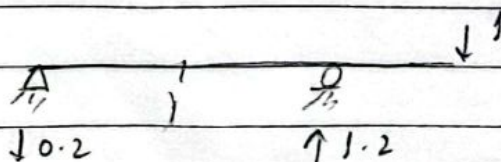
For θ_C



For Δ_D



For Δ_C



$$\rightarrow I. \theta_A = \sum \int \frac{Mm \, dx}{EI}$$

$$\theta_A = \int_0^4 \frac{(61x - 5x^2) \left(1 - \frac{x}{10}\right)}{EI} dx + \int_0^6 \frac{(59x - 5x^2 - 10) \left(\frac{x}{10}\right)}{EI} dx$$

$$= \frac{1416}{5EI} + \frac{1224}{5EI}$$

$$= \frac{528}{EI} (\curvearrowright)$$

$$\rightarrow I. \theta_B = \sum \int \frac{Mm \, dx}{EI}$$

$$= \int_0^4 \frac{(61x - 5x^2)(-0.1x)}{EI} dx + \int_0^6 \frac{(59x - 5x^2 - 10)(-1 + 0.1x)}{EI} dx$$

$$= \frac{-1472}{15EI} + \left(\frac{-1986}{5EI} \right)$$

$$= \frac{-1486}{3EI}$$

$$= \frac{1486}{3EI} (\curvearrowright)$$

$$\rightarrow I. \theta_C = \sum \int \frac{Mm \, dx}{EI}$$

$$= \int_0^4 \frac{(61x - 5x^2)(0.1x)}{EI} dx + \int_0^6 \frac{(59x - 5x^2 - 10)(1 - 0.1x)}{EI} dx$$

$$+ \int_0^2 \frac{(-5x)(1)}{EI} dx$$

$$= \frac{1472}{15EI} + \frac{1986}{5EI} - \frac{10}{EI}$$

$$= \frac{1456}{3EI} (\curvearrowright)$$

$$1. \Delta_D = \sum \int \frac{Mm \, dx}{EI}$$

$$= \int_0^4 \frac{(61x - 5x^2)(0.6x) \, dx}{EI} + \int_0^6 \frac{(59x - 5x^2 - 10)(0.4x) \, dx}{EI}$$

$$= \frac{2944}{5EI} + \frac{4896}{5EI}$$

$$= \frac{1568}{EI} \quad (\downarrow)$$

$$1. \Delta_C = \sum \int \frac{Mm \, dx}{EI}$$

$$= \int_0^4 \frac{(61x - 5x^2)(-0.2x) \, dx}{EI} + \int_0^6 \frac{(59x - 5x^2 - 10)(-0.2x) \, dx}{EI}$$

$$+ \int_0^2 \frac{(-5x)(-x) \, dx}{EI}$$

$$= \frac{-2944}{15EI} - \frac{3972}{5EI} + \frac{40}{3EI}$$

$$= \frac{-2932}{3EI}$$

$$= \frac{2932}{3EI} \quad (\uparrow)$$

Frame

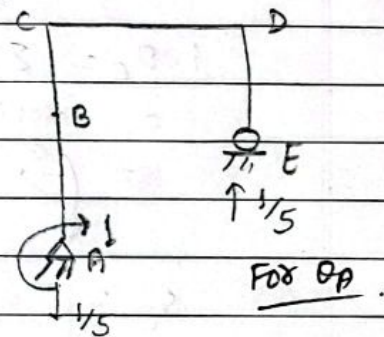
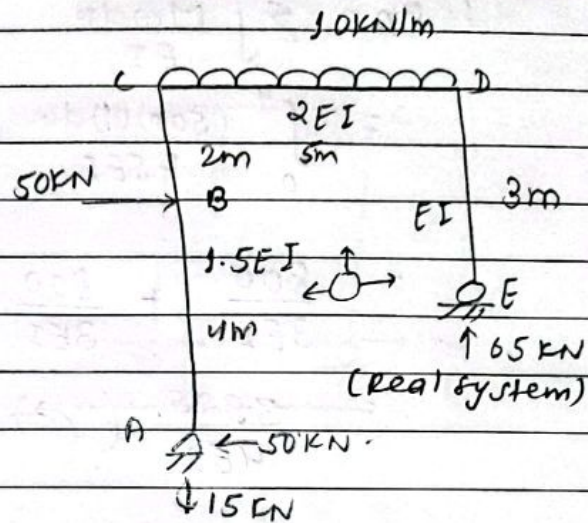
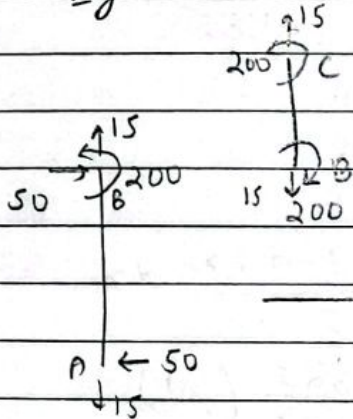
$$\theta_A = ?, (\Delta_E)_H = ?, \theta_E = ?$$

Calculation of RIR.

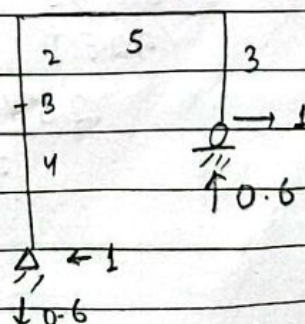
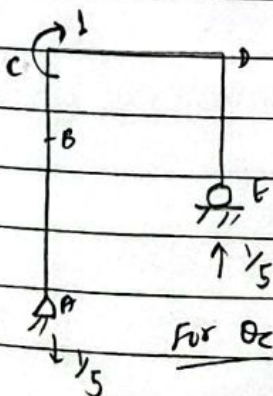
$$R_{Ax} = 50 \text{ kN} (\leftarrow)$$

$$R_{Ay} = 15 \text{ kN} (\downarrow)$$

$$R_{Ey} = 65 \text{ kN} (\uparrow)$$



Portion	AB	BC	CD	DE
Origin	A	B	C	E
Limit	0-4	0-2	0-5	0-3
EI	1.5	1.5	2	1
M	$50x$	200	$200 - 5x^2 - 15x$	0
θ_p	1	1	$1 - 0.2x$	0
θ_c	0	0	$1 - 0.2x$	0
$(\Delta_E)_H$	x	$x+4$	$-0.6x+6$	x

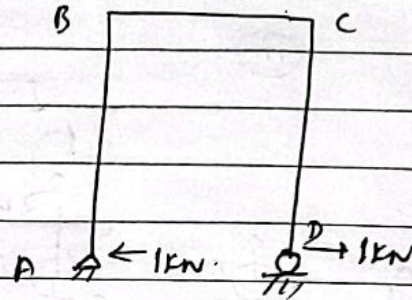
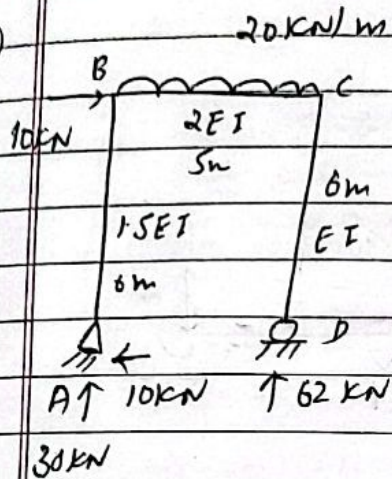


$$\begin{aligned}
 1. \theta_A &= \sum \int \frac{M m d\alpha}{EI} \\
 &= \int_0^4 \frac{(50\alpha)(1)}{1.5EI} d\alpha + \int_0^2 \frac{(200)(1)}{1.5EI} d\alpha + \int_0^5 \frac{(200-5\alpha^2)(1-0.2\alpha)}{2EI} d\alpha \\
 &= \frac{800}{3EI} + \frac{800}{3EI} + \frac{8375}{12EI} - \frac{4625}{24EI} \\
 &= \frac{3925}{4EI} (\rightarrow) = \frac{18175}{24EI} (\rightarrow) = \frac{17425}{24EI} (\rightarrow)
 \end{aligned}$$

$$\begin{aligned}
 1. \theta_C &= \sum \int \frac{M m d\alpha}{EI} \\
 &= \int_0^5 \frac{(200-5\alpha^2)(1-0.2\alpha)}{2EI} d\alpha \\
 \theta_C &= \frac{5375}{24EI} (\rightarrow) - \frac{4625}{24EI} (\rightarrow)
 \end{aligned}$$

$$\begin{aligned}
 1. (\Delta_E)_H &= \sum \int \frac{M m d\alpha}{EI} \\
 &= \int_0^4 \frac{(50\alpha)(\alpha)}{1.5EI} d\alpha + \int_0^2 \frac{(200)(\alpha)}{1.5EI} d\alpha \\
 &\quad + \int_0^5 \frac{(200-15\alpha-5\alpha^2)(-0.6\alpha+6)}{2EI} d\alpha + 0 \\
 &= \frac{6400}{9EI} + \frac{40000}{3EI} + \frac{2625}{8EI} - \frac{11875}{8EI} \\
 &= \frac{46775}{72EI} (\rightarrow) \\
 &= \frac{254075}{72EI}
 \end{aligned}$$

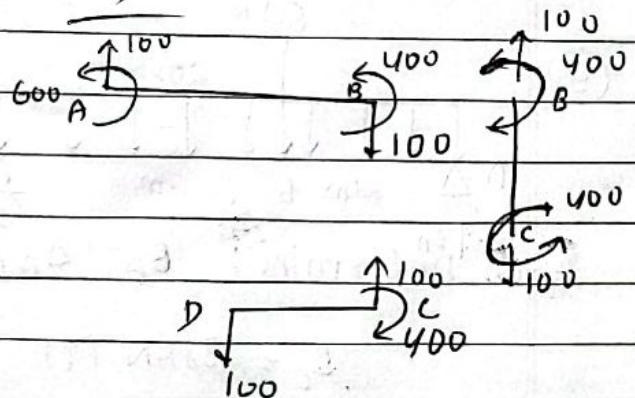
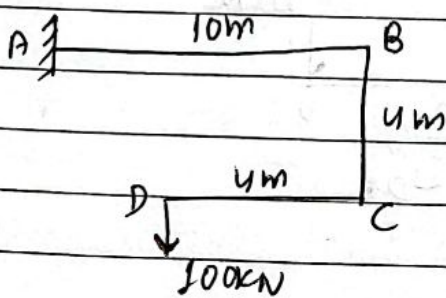
①



portion	EI	limit	origin	M	m
AB	1.5	0-6	A	$10x$	x
BC	2	0-5	C	$62x - 10x^2$	6
CD	1	0-6	D	0	x

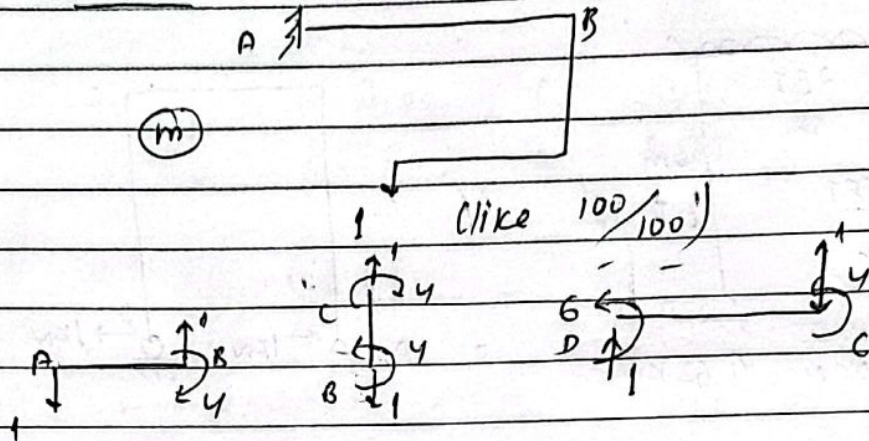
$$\begin{aligned}
 \Delta_D &= \int_0^6 \frac{(10x) x}{1.5EI} dx + \int_0^5 \frac{(62x - 10x^2) 6}{2EI} dx \\
 &= \frac{480}{EI} + \frac{1075}{EI} \\
 &= \frac{1555}{EI} (\rightarrow)
 \end{aligned}$$

②



Find vertical deflection at D.

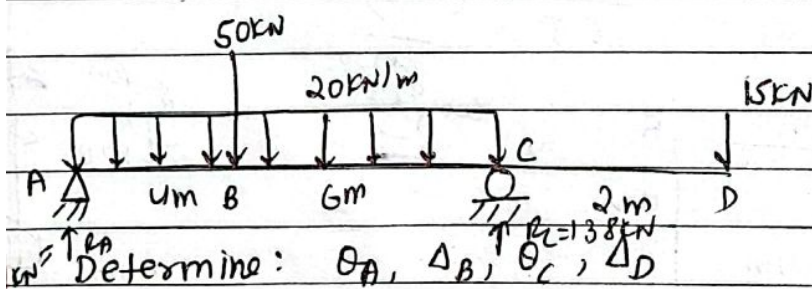
Unit Area



$$1 \cdot (\Delta_V)_D = \int_0^4 \frac{(-100x)(-x)}{EI} dx + \int_0^4 \frac{(400)(4)}{EI} dx + \int_0^{10} \frac{(400 - 10x)}{EI} dx$$

$$(\Delta_V)_D = \frac{6400}{3EI} + \frac{6400}{EI} + \frac{3500}{EI}$$

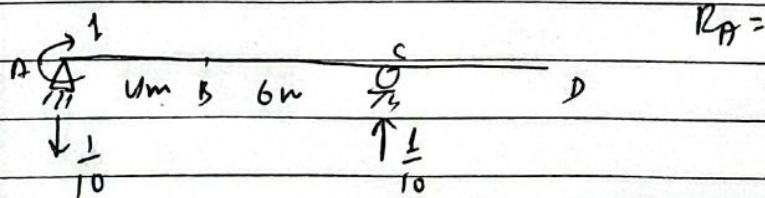
$$= \frac{36100}{3EI}$$



$$R_C = 138 \text{ kN} (\uparrow)$$

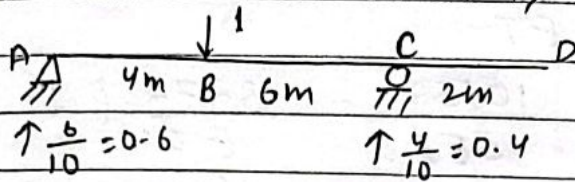
$$R_A = 127 \text{ kN} (\uparrow)$$

For unit virtual moment at A,



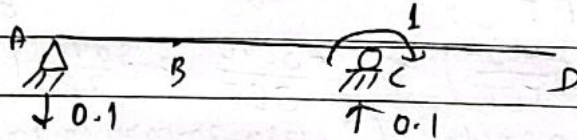
For unit virtual load at B,

(Δ_B)



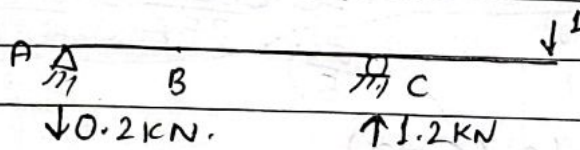
Unit moment (cw) at C,

(θ_C)



Unit vertical virtual ~~moment~~ load at D

(Δ_D)



Portion	A B	BC	B C	C D
origin	A		C	D
limit	0-4		0-6	0-2
M	$127x - 10x^2$		$138x - 15(x+2) - 10x^2$ $\hookrightarrow 123x - 30 - 10x^2$	$-15x$
(θ _B)	m	$1 - 0.1x$	$0.1x$	0
(Δ _B)	m	$0.6x$	$0.4x$	0
(θ _C)	m	$-0.1x$	$-1 + 0.1x$	0
(Δ _D)	m	$-0.2x$	$1.2x - 1(x+2)$ $\hookrightarrow 0.2x - 2$	$-x$

Using virtual load method,

$$\begin{aligned}
 \delta \cdot \theta_A &= \int \frac{M m dx}{EI} = \int_0^4 \frac{(127x - 10x^2)(1 - 0.1x) dx}{EI} \\
 &\quad + \int_0^6 \frac{(123x - 30 - 10x^2)(0.1x) dx}{EI} + 0 \\
 &= \frac{8936}{15EI} + \frac{2538}{5EI} = \frac{3310}{3EI} \quad (\checkmark)
 \end{aligned}$$

$$1. \Delta_B = \int_0^4 \frac{(127x - 10x^2)(0.6x) dx}{EI} + \int_0^6 \frac{(123x - 30 - 10x^2)(0.4x) dx}{EI}$$

$$= \frac{6208}{5EI} + \frac{10152}{5EI} - \frac{3272}{EI} \quad (\downarrow)$$

$$1. \theta_C = \int_0^4 \frac{(127x - 10x^2)(-0.1x) dx}{EI} + \int_0^6 \frac{(123x - 30 - 10x^2)(-1 + 0.1x) dx}{EI}$$

$$= \frac{-3104}{15EI} + \left[\frac{-4032}{5EI} \right] = \frac{-3040}{3EI} = \frac{3040}{3EI} \quad (\uparrow)$$

$$1. \Delta_D = \int_0^4 \frac{(127x - 10x^2)(-0.2x) dx}{EI} + \int_0^6 \frac{(123x - 30 - 10x^2)(0.2x - 2x^2) + \int_0^2 \frac{(-1x)(x) dx}{EI}}{EI}$$

$$= \frac{-6208}{3EI} = \frac{6208}{3EI} \quad (\uparrow) = \frac{5960}{3EI} \quad (\uparrow)$$

Truss

Formula:

$$1. \Delta = \int (\sigma^* dA) (E d\alpha)$$

virtual
load

real displacement

Truss $\rightarrow$ different/discrete members.

$$1. \Delta = \sum \left(\frac{P}{A} \right) (A) \left(\frac{P}{A} \right) \left(\frac{L}{E} \right)$$

stress
due to
virtual load

stress due to
real load.

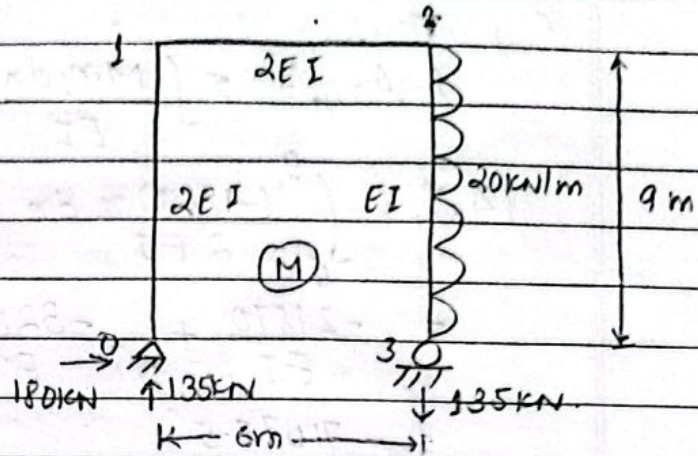
$$\left[\Delta = \sum_{i=1}^n \frac{P_i^* P_i L_i}{A_i E} \right]$$

[2071 Bhadoa]

Solution.

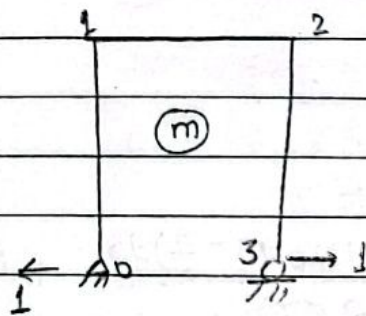
$(\Delta_3)_H = ?$

$(\theta_0) = ?$

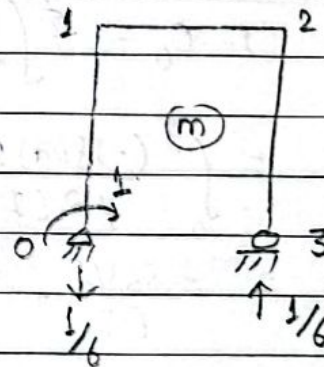


For $(\Delta_3)_H$, applying unit horizontal load at 3,

$R_3 = \frac{20 \times 9 \times 4.5}{6} = 135 \text{ kN}$



For θ_0 , applying unit moment (c/w) at 0,



Portion	0-1	1-2	2-3
limit	0-9	0-6	0-9
EI	2	2	1
origin	0	1	3
M	$-180x$	$135x - 180 \times 9$ $135x - 1620$	$-20 \times 10x^2$
m_1	x	9	x
m_2	1	$1 - \frac{x}{6}$	0

Thus,

$$1. (\Delta_3)_H = \sum \int \frac{M m_1 dx}{EI}$$

$$(\Delta_3)_H = \int_0^9 \frac{(-180x)x}{2EI} dx + \int_0^6 \frac{(135x-1620)9}{2EI} dx + \int_0^9 \frac{(-10x^2)x}{EI} dx$$

$$= \frac{-21870}{EI} + \frac{-32805}{EI} + \frac{-32805}{2EI}$$

$$= \frac{-71077.5}{EI}$$

$$= \frac{71077.5}{EI} (\leftarrow)$$

Also,

$$1. \theta_0 = \sum \int \frac{M m_2 dx}{EI}$$

$$= \int_0^9 \frac{(-180x)1}{2EI} dx + \int_0^6 \frac{(135x-1620)(1-\frac{x}{6})}{2EI} dx + 0$$

$$= \frac{-3645}{EI} + \frac{-2025}{EI}$$

$$= \frac{-5670}{EI}$$

$$= \frac{5670}{EI} (\rightarrow)$$

[2064 POWH]

→

20/2

A, B, C → pin jointed

AB → R.C.C. beam

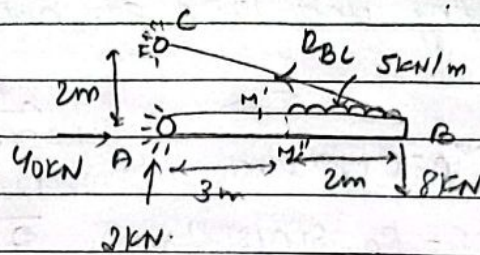
↳ 250mm breadth
↳ 500mm depth

BC → steel

↳ 10mm dia.

$$E_c = 1 \times 10^6 \text{ kg/cm}^2, E_s = 2 \times 10^6 \text{ kg/cm}^2$$

↳ vertical deflection at middle of beam due to bending.



$$\sum M_B = 0 (\curvearrowright +ve)$$

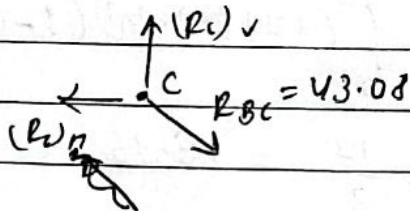
$$\text{on } R_A \times 5 - 5 \times 2 \times 1 = 0$$

$$\Rightarrow [R_A = 2 \text{ kN}(\uparrow)]$$

$$\sum M_A = 0 (\curvearrowright +ve)$$

$$\text{on } 5 \times 2 \times 4 + 8 \times 5 - R_{BC} \sin(\tan^{-1}(\frac{2}{5})) \times 5 = 0$$

$$\Rightarrow [R_{BC} = 43.08 \text{ kN}]$$



$$(R_c)_v = 43.08 \sin(\tan^{-1}(\frac{2}{5}))$$

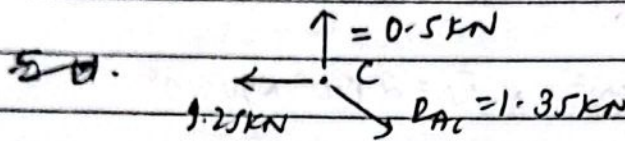
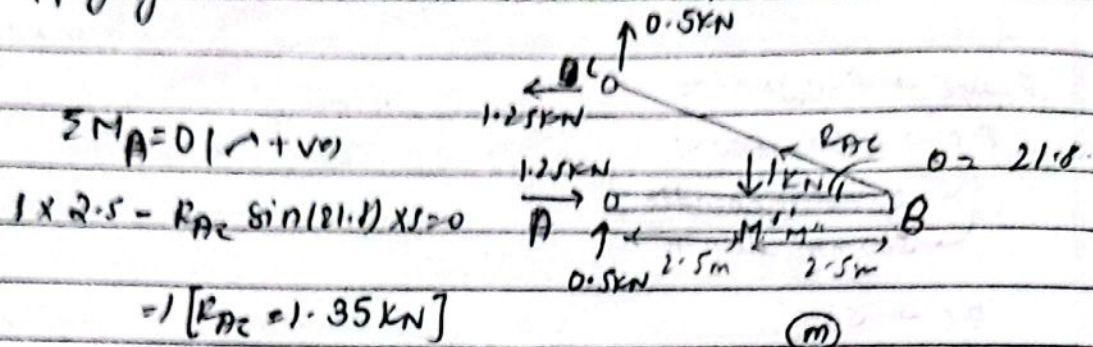
$$(R_c)_v = 16 \text{ kN}(\uparrow)$$

$$(R_c)_h = 43.08 \cos(\tan^{-1}(\frac{2}{5}))$$

$$= 40 \text{ kN}(\leftarrow)$$

$$\text{Thus, } (R_A)_h = 40 \text{ kN}(\rightarrow)$$

Applying unit load at middle of beam



Position	Limit	Origin	M	m
A-M'	0-2.5	A	$2x$	$0.5x$
M'-M''	0-0.5	M'	$2(x+2.5)$	$0.5(x+2.5) - x$
M''-B	0-2	M''	$2(x+3) - 2.5x^2$	$0.5(x+3) - (x+0.5)$

$$EI \times \delta \Delta = \int_0^{2.5} (2x)(0.5x) dx + \int_0^{0.5} (2x+5)(1.25-0.5x) dx + \int_0^2 (2x+6-2.5x^2)(1-0.5x) dx$$

$$= \frac{125}{24} + \frac{37}{12} + \frac{17}{3} = 13.96$$

$$\Delta = \frac{13.96 \times 1000}{2 \times 10^6 \times 10^4 \times 0.25 \times 0.5^3} = 0.0273 \text{ mm}$$

TRUSS

Deflection due to applied load

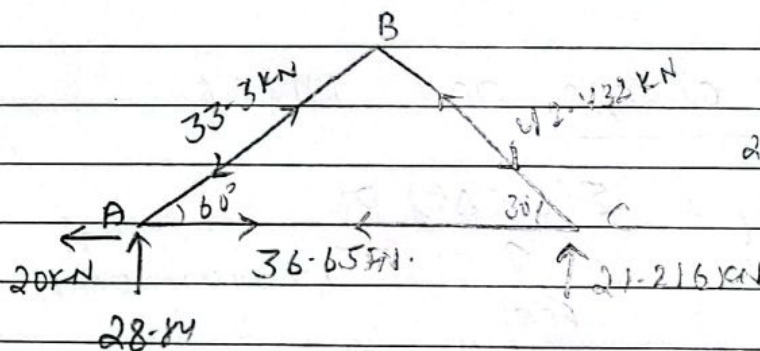
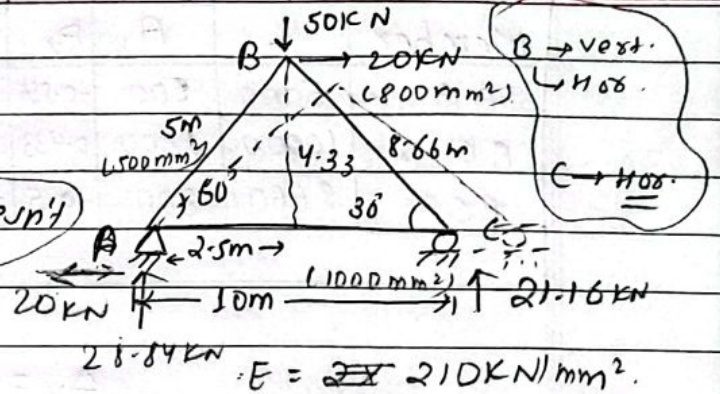
$$\Delta = \frac{\sum p_i p_i L_i}{A_i E}$$

A is hinged. so, it doesn't move.

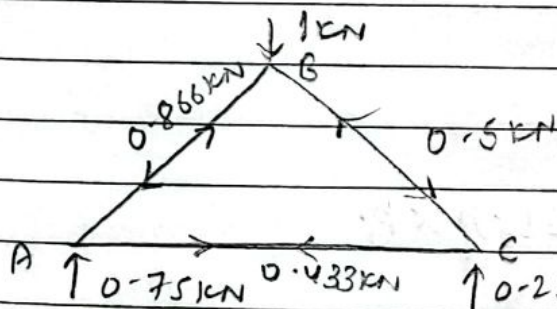
$$\Delta B_V = ?$$

$$\Delta B_H = ?$$

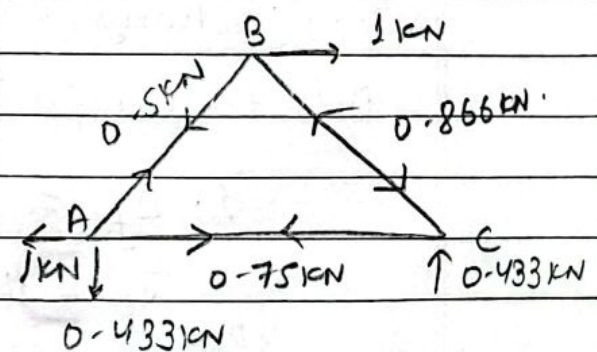
First $p_i \rightarrow P_i$ } so, that if p_i is 0 for any member no need to find P_i for that member.



For unit vertical load at B.



For unit



$$\Delta B_V = \frac{(-33.3)(-0.866) \times 5000}{500 \times 210} + \frac{(36.65)(0.433) \times 10000}{1000 \times 210} + \frac{(-42.432)(-0.5) \times 8660}{800 \times 210}$$

$$= 3.22 \text{ mm } (\downarrow)$$

$$\Delta B_H = \frac{(-33.3)(0.5) \times 5000}{500 \times 210} + \frac{(36.65)(0.75) \times 10000}{1000 \times 210} + \frac{(-42.432)(-0.866) \times 8660}{800 \times 210}$$

$$= 2.41 \text{ mm } (\rightarrow)$$

Member	L	A	P_V	P_H	P	$\frac{P_V PL}{A}$	$\frac{P_H PL}{A}$
AB	5000	500	-0.866	0.5	-33.30	288.38	-166.5
AC	10000	1000	0.433	0.75	36.65	158.69	274.875
BC	8660	800	-0.5	-0.866	-42.32	229.057	396.73
$\Sigma = 6786.227$						$\Sigma = 505.105$	

$$\Delta_V = \frac{\Sigma_1}{E} = 3.22 \text{ mm}$$

$$\Delta_H = \frac{\Sigma_2}{E} = 2.41 \text{ mm}$$

Temperature change and Misfit

$$\downarrow \quad \uparrow \quad \swarrow \quad \searrow$$

$$1. \Delta_t = \frac{\Sigma (L \alpha \Delta t) P_i}{E}$$

due to temperature change real displacement Internal virtual load

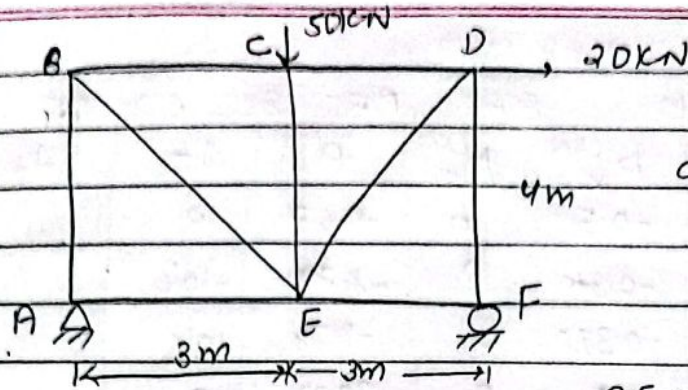
Due to misfit

$$\downarrow$$

$$1. \Delta_f = \frac{\Sigma (\Delta L_f)_i \times P_i}{E}$$

due to misfit

(9)



Temperature of AE and EF raised by 20°C
BC and CD falls by 10°C .

$$\alpha = 1.2 \times 10^{-6}/^\circ\text{C}$$

$$E = 210 \text{ kN/mm}^2$$

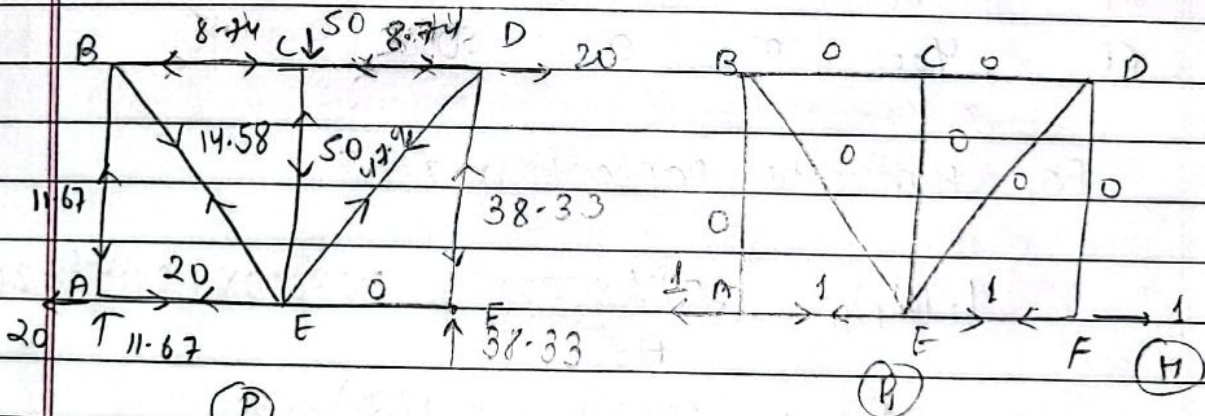
Area for all

$$A = 500 \text{ mm}^2$$

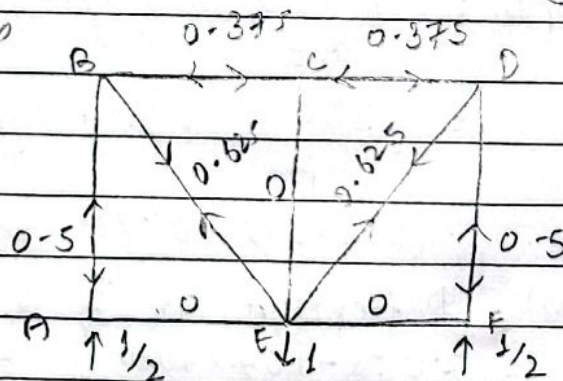
BE is too short by 5mm
DE is too long by 10mm

$$(\Delta F)_H = ?$$

$$(\Delta E)_V = ?$$

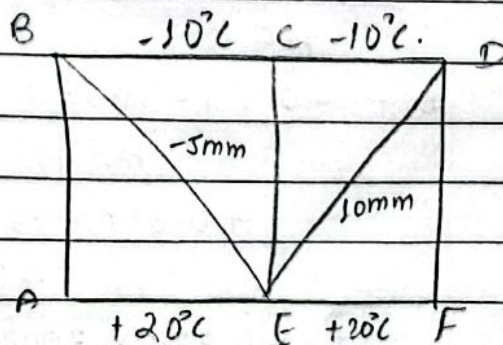


(P)
(upper members
↓ comp
lower
↑ tension)



BE WD = AB

(P₂) (V)



Member	AE	EF	AB	BC	CD	DE
Member	L(mm)	$P_v^{(KN)}$	$P_H^{(KN)}$	$P^{(KN)}$	Δt	Δf
AB	4000	-0.5	0	-11.67	0	
BC	3000	-0.375	0	-8.74	-10°C	
CD	3000	-0.375	0	-8.74	-10°C	
DE	4000	-0.5	0	-38.33	0	
AE	3000	0	1	20	+20°C	
EF	3000	0	1	0	+20°C	
BE	5000	+0.625	0	14.58	0	-5mm (too short)
DE	5000	+0.625	0	47.9	0	+10mm (too long)
CE	4000	0	0	-50	0	

For, Horizontal Deflection at F,

$$1. (\Delta_F)_H = \frac{20 \times 1 \times 3000}{AE} + 1 \times 3000 \times 1.2 \times 10^{-6} \times 20 \times 2$$

$$(\Delta_F)_H = \left(\frac{60000}{AE} + \frac{0.072}{\times 2} \right) \text{mm.}$$

$$= \left(\frac{285.71}{A} + 0.072 \right) \text{mm}$$

$$= (0.57142 + 0.072) = 0.643 \text{mm} \cdot 0.715 \text{mm}$$

For vertical Deflection at ^{to}E,

$$1. (\Delta_E)_V = \frac{-0.5 \times -11.67 \times 4000}{AE} + \frac{-0.375 \times -8.74 \times 3000 \times 2}{AE}$$

$$+ \frac{-0.5 \times -38.33 \times 4000}{AE} + \frac{0.625 \times 14.58 \times 5000}{AE}$$

$$+ \frac{0.625 \times 47.9 \times 5000}{AE}$$

$$+ \frac{(-0.375) \times 3000 \times 1.2 \times 10^{-6} \times (-10) \times 2}{AE}$$

$$+ \frac{(0.625) \times (-5) + (0.625) \times 10}{AE}$$

$$(\Delta_E)_V = \left(\frac{314915}{AE} + 0.027 + 3.125 \right)$$

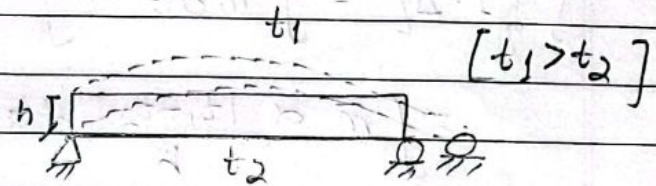
$$(\Delta_E) v = \left(\frac{314915}{AE} + 3.152 \right) \text{ mm.}$$

$$= \frac{(1499.6 + 3.152)}{A} \text{ mm.}$$

$$= 3 + 3.152$$

$$= \underline{6.152 \text{ mm}}$$

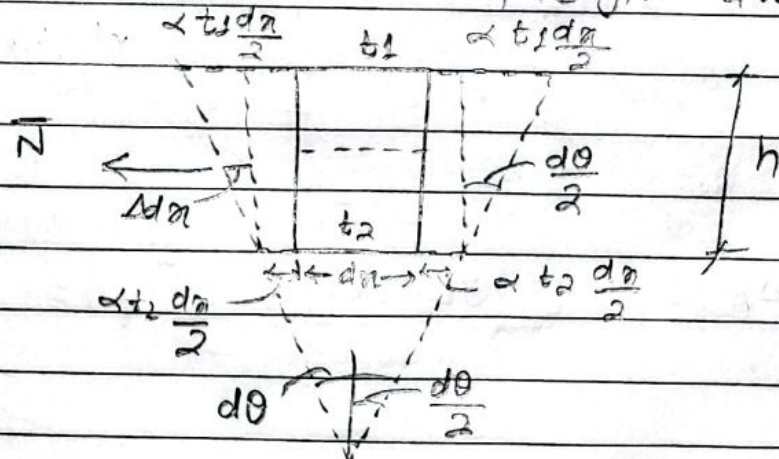
Beam Frame



$$\delta \Delta_t = \frac{\alpha (t_1 - t_2)}{h} \int \bar{m} \, dx + \frac{\alpha (t_1 + t_2)}{2} \int \bar{N} \, dx$$

$\uparrow$ due to bending moment $\uparrow$ due to axial force

Take small element of length 'dx'.



$$\frac{d\theta}{2} = \frac{\alpha t_1 \frac{dx}{2}}{h} - \frac{\alpha t_2 \frac{dx}{2}}{h}$$

$$d\theta = \frac{\alpha (t_1 - t_2) dx}{h}$$

(change in length in center line)

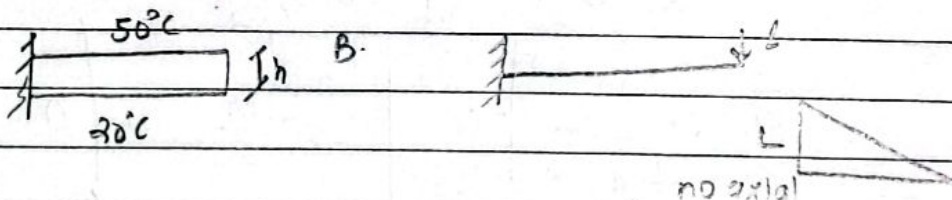
$$\Delta d\alpha = \text{both side } \left(\frac{\alpha t_1 d\alpha}{2} + \alpha \frac{t_2 d\alpha}{2} \right)$$

Avg. of change in upper & lower part.

Now,

$$\begin{aligned} \int \int 1 \cdot \Delta t &= \int \bar{m} d\alpha + \int \bar{N} (\Delta d\alpha) \\ &= \int \bar{m} \alpha \frac{|t_1 - t_2|}{h} d\alpha + \int \bar{N} \alpha \frac{|t_1 + t_2|}{2} d\alpha \\ &= \frac{\alpha |t_1 - t_2|}{h} \int \bar{m} d\alpha + \frac{\alpha |t_1 + t_2|}{2} \int \bar{N} d\alpha \\ &= \frac{\alpha |t_1 - t_2|}{h} \underbrace{\int \bar{m} d\alpha}_{\text{Area under } \bar{m} \text{ dia}} + \frac{\alpha |t_1 + t_2|}{2} \underbrace{\int \bar{N} d\alpha}_{\text{Area under } \bar{N} \text{ dia}} \end{aligned}$$

(9)

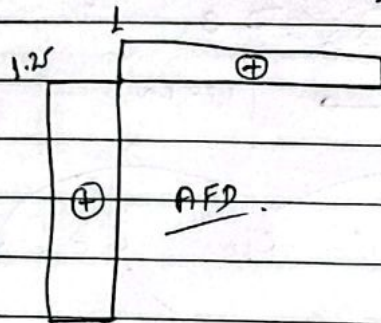
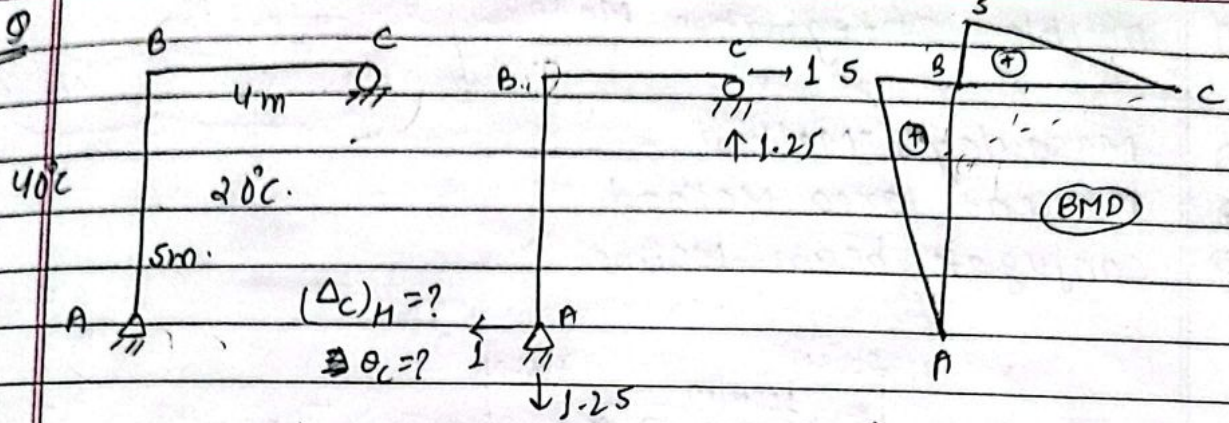


$$(\Delta B)_v = \frac{\alpha (50 - 20)}{h} \times \frac{L^2}{2} + 0 = \frac{30\alpha L^2}{2h} = \frac{15\alpha L^2}{h}$$

✓ If tension due to temperature variation and unit vertical load is on same side take +ve sign.

If they are on opposite side → take -ve sign.

tension → where temp. is larger.



$$(\Delta_c)_H = \left\{ (\Delta_c)_H \right\}_\downarrow + \left[\frac{-\alpha \times (40-20)}{h} \times \frac{1}{2} \times 5 \times 5 \right] + \left[\frac{-\alpha \times (40-20)}{h} \times \frac{1}{2} \times 5 \times 4 \right] + \left[\frac{\alpha \times (40+20)}{2} \times 5 \times 1.25 \right] + \left[\frac{\alpha \times (40+20)}{2} \times 4 \times 1 \right]$$

avg. temp

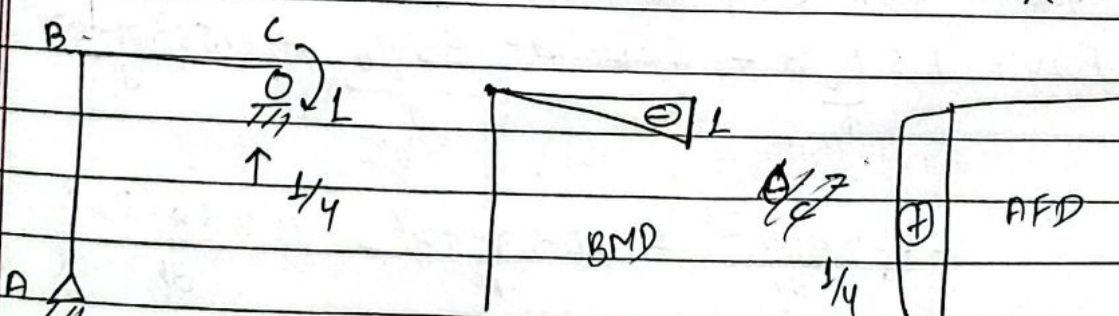
(+ve)

Tension

$$= \frac{-450\alpha}{h} + \frac{307.5\alpha}{h} \quad \leftarrow (-)$$

$$= \frac{-757.5\alpha}{h} \quad \leftarrow 182.5\alpha$$

$$= \frac{757.5\alpha}{h} \quad \leftarrow 182.5\alpha$$



$$\theta_c = \frac{\alpha \times (40-20)}{h} \times 0 + \left(\frac{\alpha \times (40-20)}{h} \times \frac{1}{2} \times 1 \times 4 \right) + \frac{\alpha \times (40+20)}{2h} \times \frac{1}{4} \times 5$$

Chapter 8-4

Deflection of Beam

- ① Double Integration Method

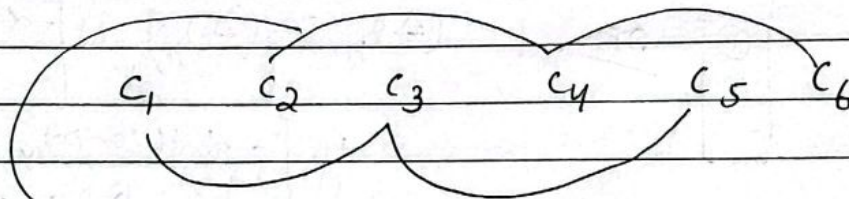
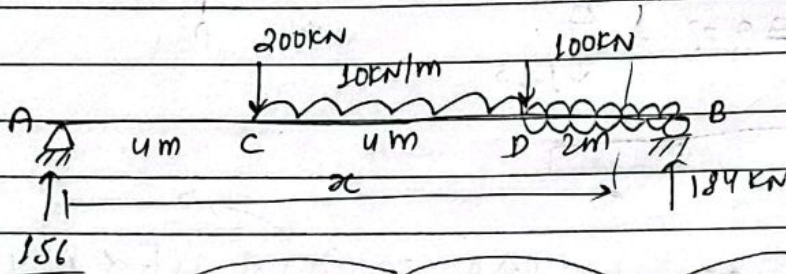
$$EI \frac{d^2y}{dx^2} = M_x$$

$C_1 \quad C_2 \quad C_3 \quad C_4 \quad C_5$

- ② Macaulay's Method

- ③ Moment area Method

- ④ conjugate beam Method.



$$EI \frac{d\theta}{dx}$$

equal = slope at origin
deflection at origin

$$M_x = 156 \langle x-0 \rangle^1 - 200 \langle x-4 \rangle^1 - 10 \langle x-4 \rangle^2 - 100 \langle x-8 \rangle^1 + 10 \langle x-8 \rangle^2$$

Equation of slope:-

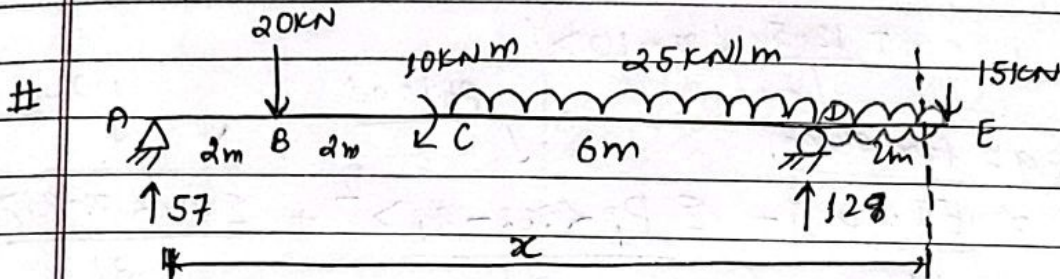
$$EI\theta = EI\theta_0 + \frac{156 \langle x-0 \rangle^2}{2!} - \frac{200 \langle x-4 \rangle^2}{2!} - \frac{10 \langle x-4 \rangle^3}{3!} - \frac{100 \langle x-8 \rangle^2}{2!} + \frac{10 \langle x-8 \rangle^3}{3!} \quad (\text{downward}) \quad \text{--- (I)}$$

Equation for deflection

$$EIy = EI\theta_0 x + \frac{156 \langle x-0 \rangle^3}{3!} - \frac{200 \langle x-4 \rangle^3}{3!} - \frac{10 \langle x-4 \rangle^4}{4!} - \frac{100 \langle x-8 \rangle^3}{3!} + \frac{10 \langle x-8 \rangle^4}{4!} \quad \text{--- (II)}$$

For, $x=0, y=0$
 $[EI y_0 = 0]$

For $x=10, y=0$ ← support no def.
 $EI \theta_0 = -1813.3333$



→ Solution.

$$EI \frac{d^2 y}{dx^2} = M_x = 57 \langle x-0 \rangle^1 - 20 \langle x-2 \rangle^1 + 10 \langle x-4 \rangle^0 - 12.5 \langle x-4 \rangle^2 + 128 \langle x-10 \rangle^1 + 12.5 \langle x-10 \rangle^2 \quad \text{--- (I)}$$

$$\text{or, } EI \frac{dy}{dx} = \theta_x = \frac{57}{2} \langle x-0 \rangle^2 + C_1 - \frac{20}{2} \langle x-2 \rangle^2 + 10 \langle x-4 \rangle^1 - \frac{12.5}{3} \langle x-4 \rangle^3 + \frac{128}{2} \langle x-10 \rangle^2 + \frac{12.5}{3} \langle x-10 \rangle^3 \quad \text{--- (II)}$$

$$\text{or, } EI y = \frac{57}{6} \langle x-0 \rangle^3 + C_1 x + C_2 - \frac{20}{6} \langle x-2 \rangle^3 + \frac{10}{2} \langle x-4 \rangle^2 - \frac{12.5}{12} \langle x-4 \rangle^4 + \frac{128}{6} \langle x-10 \rangle^3 + \frac{12.5}{12} \langle x-10 \rangle^4 \quad \text{--- (III)}$$

Using Boundary condition,

For, $x=0, y=0, C_2 = 0$

For, $x=10, y=0, C_1 = -\frac{1987}{3}$

Thus, Eqⁿ for slope,

$$EI \frac{dy}{dx} = \frac{57}{2} \langle x-0 \rangle^2 - \frac{1987}{3} - \frac{20}{2} \langle x-2 \rangle^2 + 10 \langle x-4 \rangle^1 - \frac{12.5}{3} \langle x-4 \rangle^3 + \frac{128}{2} \langle x-10 \rangle^2 + \frac{12.5}{3} \langle x-10 \rangle^3$$

Eqⁿ for deflection,

$$EI y = \frac{57 \langle x-0 \rangle^3}{6} - \frac{1987 x}{3} - \frac{20 \langle x-2 \rangle^3}{6} + \frac{10 \langle x-4 \rangle^2}{2} - \frac{12.5 \langle x-4 \rangle^4}{12} + \frac{128 \langle x-10 \rangle^3}{6} + \frac{12.5 \langle x-10 \rangle^4}{12}$$

'OR' Eqⁿ for slope

$$EI \theta = \underbrace{EI \theta_0}_{\text{III}} + \underbrace{\sum P_i \langle x - a_i \rangle^2}_{\text{2I}} + \underbrace{\sum w_i \langle x - a_i \rangle^3}_{\text{3I}} + \underbrace{\sum M_i \langle x - a_i \rangle^1}_{\text{moment}}$$

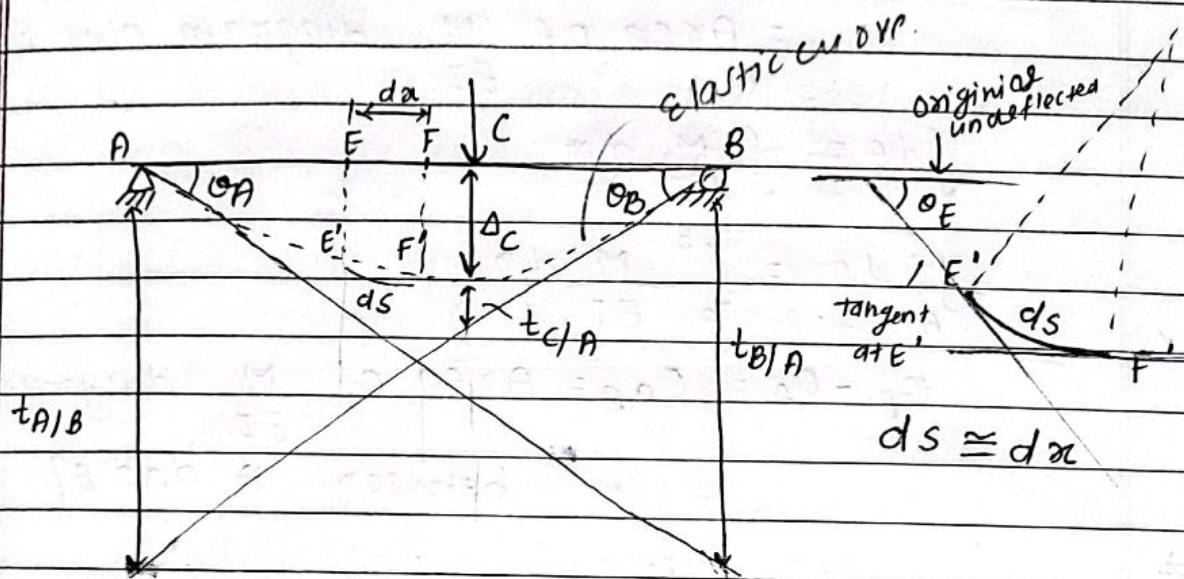
-662.3
C₁

Eqⁿ for deflection

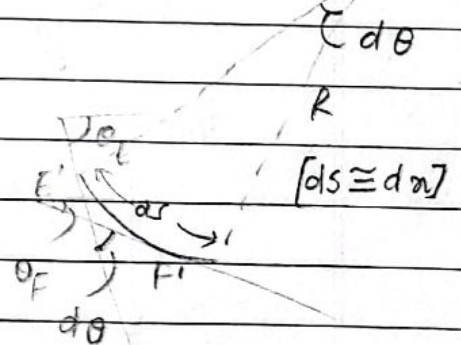
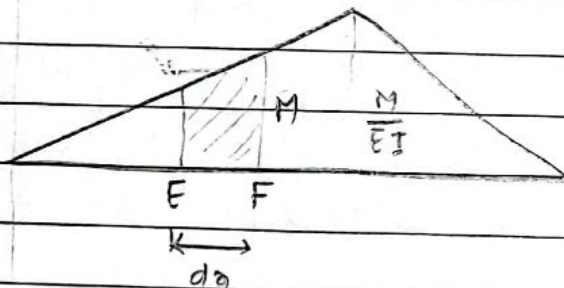
$$EI y = (EI \theta_0) x + \underbrace{EI y_0}_{\text{def. at origin}} + \underbrace{\sum P_i \langle x - a_i \rangle^3}_{\text{3I}} + \underbrace{\sum w_i \langle x - a_i \rangle^4}_{\text{4I}} + \underbrace{\sum M_i \langle x - a_i \rangle^2}_{\text{2I}}$$

3. Moment Area Method

Two Theorems:-



$t_{B/A}$ = tangential deviation of B with respect to tangent at A.



$$\left[d\theta = \frac{dx}{R} = \frac{M}{EI} dx \right]$$

Mathematically,

$$ds = R d\theta$$

$$d\theta = \frac{ds}{R}$$

Theorem 1: [Area of $\frac{M}{EI}$ diagram b/w E & F].

The difference in slope between two points tangents drawn at any two points in the elastic curve is given by the Area of the $\frac{M}{EI}$ diagram between those two points.

$$\left[\frac{M}{EI} \left(d\theta = \frac{M}{EI} dx \right) \right]$$

$d\theta$ → Angle b/w tangent drawn at E' & F'

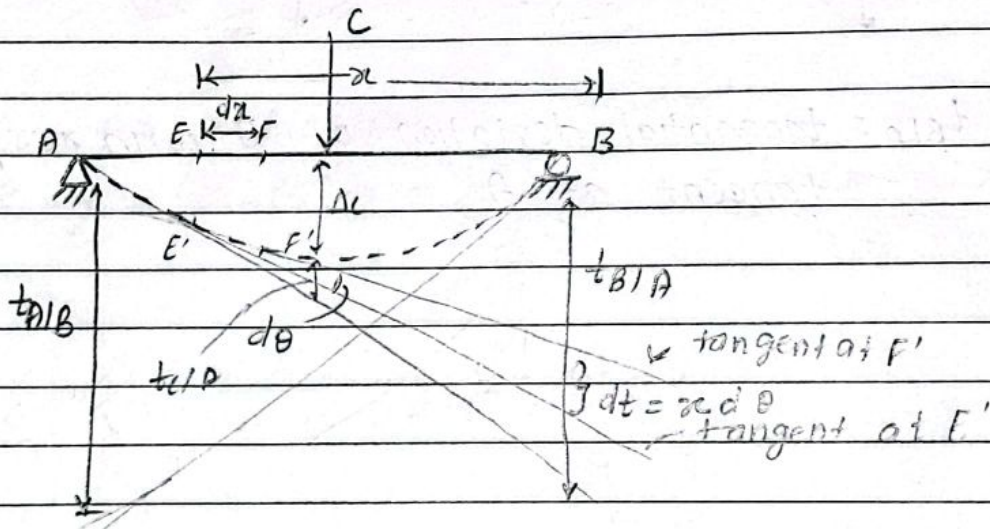
$$\left[\frac{\sigma}{y} = \frac{M}{I} = \frac{E}{R} \right]$$

$$[d\theta = \frac{dx}{R} = \frac{M}{EI} dx]$$

= Area of $\frac{M}{EI}$ diagram b/w E and F

$$\int d\theta = \int \frac{M}{EI} dx$$

$$\int_0^B d\theta = \int_0^B \frac{M}{EI} dx$$

$$\theta_B - \theta_A = \theta_{AB} = \text{Area of } \frac{M}{EI} \text{ diagram between A and B}$$


$$dt = a d\theta = \left(\frac{M}{EI} dx \right) a \quad [x \rightarrow \text{measured about } B].$$

$$= a dA$$

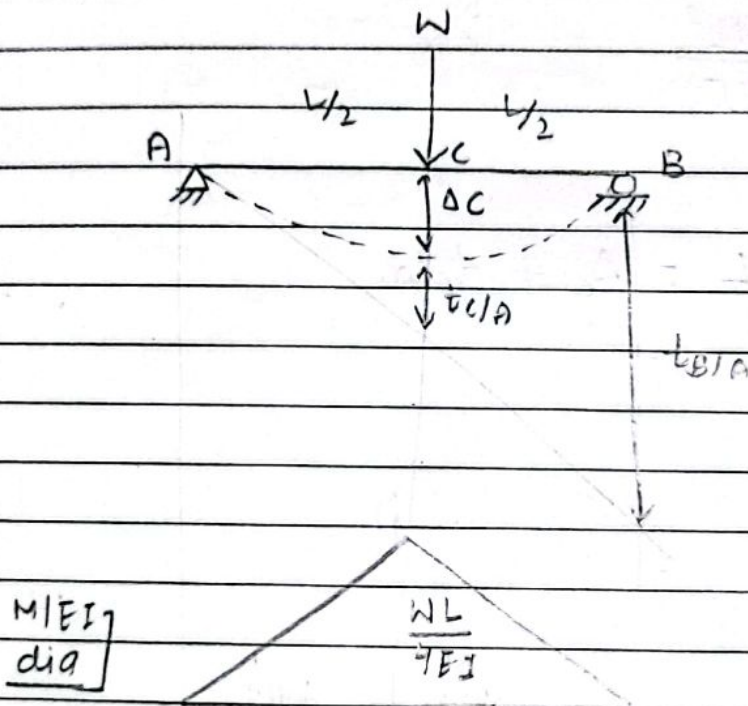
$$t_{B/A} = \int_A^B \pi \, dA$$

= Moment of M/EI diagram between A and B about B.

Theorem 2:

Moment of $\frac{M}{EI}$ diagram between two points A & B

about B gives the tangential deviation of point B with respect to tangent drawn at A. This is theorem 2.



Using Theorem 2:

$$t_{B/A} = \frac{1}{2} \times L \times \frac{WL}{4EI} \times \frac{L}{2} = \frac{WL^3}{16EI}$$

$$\left[\theta_A = \frac{t_{B/A}}{L} = \frac{WL^2}{16EI} \right]$$

$$\theta_B - \theta_A = (\text{Area})_{AB} \quad (\text{Theorem 1})$$

$$\text{or, } \theta_B - \left(-\frac{WL^2}{16EI} \right) = \frac{1}{2} \times L \times \frac{WL}{4EI}$$

$$\Rightarrow \left[\theta_B = \frac{WL^2}{16EI} \right]$$

to find Δ_c ,

$t_{C/A} = \text{Moment of } \frac{M}{EI} \text{ dia b/w A and C}$
about C

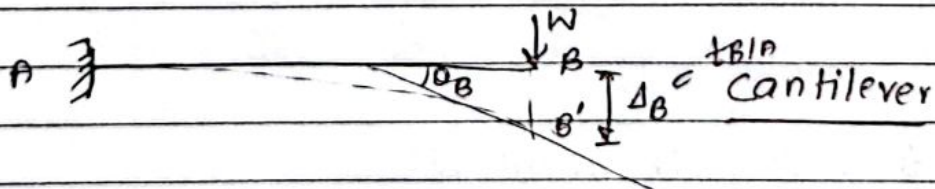
$$= \left(\frac{1}{2} \times \frac{L}{2} \times \frac{WL}{4EI} \right) \times \frac{1}{3} \left(\frac{L}{2} \right)$$

$$= \frac{WL^2}{96EI}$$

Also,

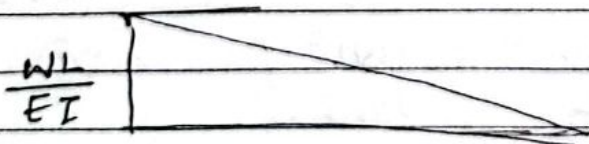
$$\theta_A \times \frac{L}{2} = \Delta_c + t_{C/A}$$

eg:



$$\theta_A = 0$$

$t_{A/B} = \text{tangential}$
deviation of from
B to tangent drawn
at A.



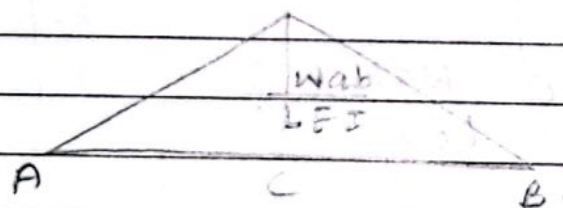
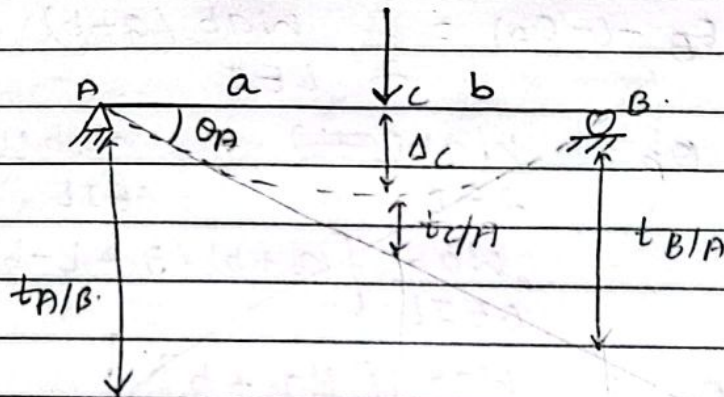
$$\theta_B - \theta_A = \text{Area under } \theta \text{ diagram}$$

$$\theta_B - 0 = \frac{1}{2} \times L \times \frac{WL}{EI}$$

$$\left[\theta_B = \frac{WL^2}{2EI} \right]$$

$$\Delta_B = t_{B/A} = \frac{1}{2} \times \frac{WL}{EI} \times L \times \frac{2L}{3} = \frac{WL^3}{3EI}$$

eg¹ Unsymmetrical load.



$$t_{B/A} = \frac{1}{2} b \times \frac{wab}{LEI} \times \frac{2b}{3} + \frac{1}{2} a \times \frac{wab}{LEI} \times \frac{(b+a)}{3}$$

$$= \frac{wab}{6LEI} [2b^2 + 3ab + a^2]$$

$$= \frac{wab}{6LEI} [(a+b)^2 + b^2 + ab]$$

$$= \frac{wab}{6LEI} [L^2 + b^2 + ab]$$

$$= \frac{wab}{6LEI} [L^2 + bL] \quad [a+b=L]$$

$$t_{B/A} = \frac{wab}{6EI} [L+b]$$

$$\theta_A = \frac{t_{B/A}}{L} = \frac{Wab(L+b)}{6EI} \quad (\curvearrowright)$$

$$\theta_B - \theta_A = \theta_{AB} = \text{Area AB} \quad \{C/W \rightarrow -ve\}$$

$$\text{or } \theta_B - (-\theta_A) = \frac{1}{2} \frac{Wab(a+b)}{LEI}$$

$$\theta_B = \frac{Wab(a+b)}{2EI} - \frac{Wab(L+b)}{6EI}$$

$$= \frac{Wab}{6EI} [(a+b) \times 3 - L - b]$$

$$\left\{ \theta_B = \frac{Wab}{6EI} [2a+b] \right\} \checkmark$$

$$t_{C/A} = \frac{1}{2} a \frac{Wab}{LEI} \times \frac{a}{3}$$

$$\theta_A \times a = \Delta_c \times t_{C/A}$$

$$\Delta_c = \frac{Wa^2b^2}{3EI}$$

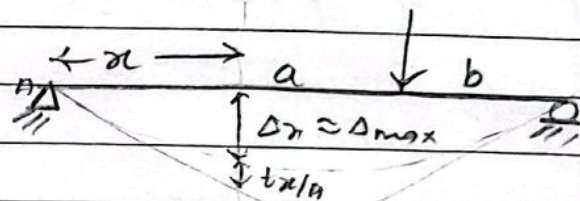
- TO find maximum deflection:

Maximum deflection will occur on larger span AC.

Let, x be the distance of max^m deflection from A.

$$\theta_x = 0 \text{ (horizontal)}$$

→ max^m deflection (say)



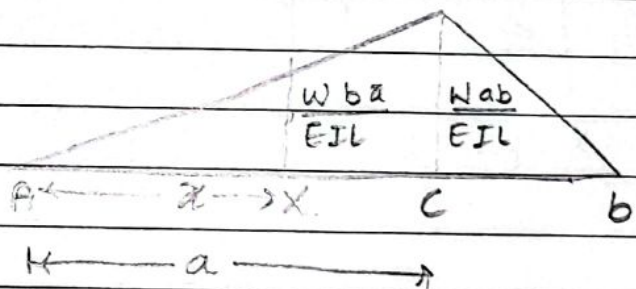
$$\theta_x - \theta_A = \theta_A x \quad \leftarrow \theta_A$$

$$\text{or, } \theta_A x = \frac{Wab(L+b)}{6EI}$$

$$\text{or, } \frac{1}{2} \times x \times \frac{Wbx}{EI} = \frac{Wab(L+b)}{6EI}$$

$$\text{or, } x^2 = \frac{a(L+b)}{3}$$

position $\left\{ x = \sqrt{\frac{a(L+b)}{3}} \right\}$



$$\tan \theta_A = \theta_A = \frac{\Delta_x + \frac{1}{2} x \times \frac{Wbx}{EI}}{x}$$

Since, $\Delta_{max} = \Delta_x$

$$[x \cdot \theta_A = \Delta_{max} + \frac{1}{2} x \times \frac{Wbx}{EI}] \quad \leftarrow$$

$$\text{or } x \cdot \frac{Wab(L+b)}{6EI} = \Delta_{max} + \frac{1}{2} \times \frac{Wbx}{EI} \times x \times \frac{x}{3}$$

$$\begin{aligned} \Delta_{max} &= \frac{Wab}{6EI} \times x(L+b) - \frac{Wbx^3}{6EI} \\ &= \frac{Wba}{6EI} [a(L+b) - x^2] \end{aligned}$$

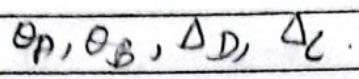


Diagram of a beam AB of length 5m. The beam is supported by a pin at A and a roller at B. A uniformly distributed load of $125/EI$ is applied downwards. The beam is labeled M/EI at the bottom. The deflection curve is shown above the beam. The equation $M_D = 50x - 5x^2$ is written on the right.

 CamScanner

For AD portion,

$$A_{AD} = \int_0^4 \frac{(50x - 50x^2)}{EI} dx$$

$$= \frac{880}{3EI}$$

$$(\text{from A}) \bar{x} = \frac{\int x dA}{A}$$

$$= \frac{\int_0^4 (50x - 50x^2) x dx}{293.33}$$

$$= 2.545m$$

$$= 1.45m (\text{from D})$$

$$\theta_A = \frac{t_{B/A}}{10} = \frac{1250}{3EI} (\curvearrowright)$$

$$\theta_A - \theta_B = \Delta_{AB} = \int_0^{10} \frac{(50x - 50x^2)}{EI} dx$$

$$\text{on } \frac{1250}{3EI} - \theta_B = \frac{2500}{3EI}$$

$$\theta_B = \frac{1250}{3EI} (\curvearrowleft)$$

$t_{D/A}$ = Moment of M/EI diagram b/w A & D abt. D

$$= \frac{880}{3EI} \times 1.45 = \frac{1276}{3EI}$$

$$\frac{\Delta_D + t_{D/A}}{t_{B/A}} = \frac{4}{10}$$

$$\Rightarrow \Delta_D = \frac{3724}{3EI} (\downarrow)$$

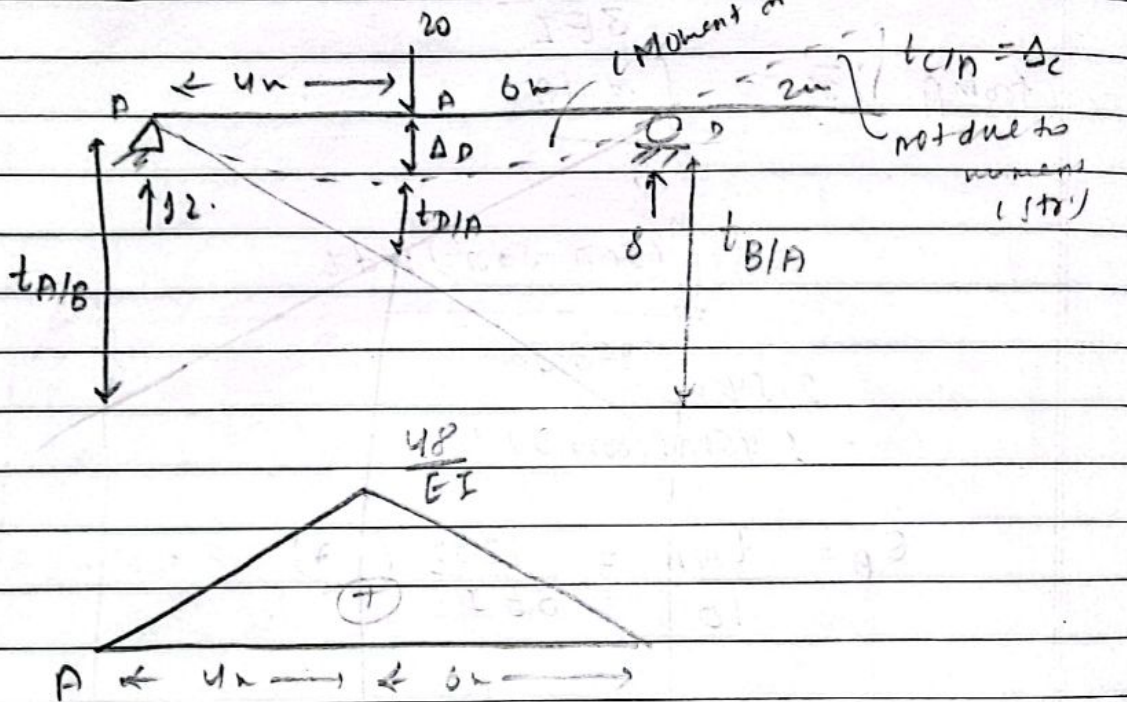
$$t_{C/A} = \text{Moment of } M/EI \text{ dia b/w A \& C}$$

$$= \left(\int_0^{10} \frac{(50x - 5x^2)}{EI} dx \right) \times (2+5)$$

$$\Rightarrow \theta_B = \frac{t_{C/A}}{2}$$

$$\Rightarrow t_{C/A} = \Delta_C = \frac{2500}{3EI} (\uparrow)$$

For 20 kN,



$[\theta_D \neq 0; \text{Here deflection won't be max}^m \text{ at D due to unsymmetricality}]$

$$\theta_A - \theta_D = \left(\frac{48}{EI} \right) \times \frac{1}{2} \times 4$$

$$\Rightarrow \left[\theta_A = \frac{96}{EI} \right] (\curvearrowright)$$

$$\theta_A - \theta_B = \left(\frac{48}{EI} \right) \times \frac{1}{2} \times 4 + \left(\frac{48}{EI} \right) \times \frac{1}{2} \times 6$$

$$\Rightarrow \left[\theta_B = \frac{144}{EI} \right] (\uparrow)$$

$$\left[t_{D/A} = \left(\frac{48}{EI} \right) \times \frac{1}{2} \times 4 \times \frac{4}{3} = \frac{128}{EI} \right] \curvearrowright$$

$$t_{B/A} = \left(\frac{48}{EI} \right) \times \frac{1}{2} \times 4 \times \left(6 + \frac{4}{3} \right) + \left(\frac{48}{EI} \right) \times \frac{1}{2} \times 6 \times \left(\frac{2 \times 6}{3} \right)$$

$$= \frac{1280}{EI}$$

$$\frac{\Delta_D + t_{D/A}}{t_{B/A}} = \frac{4}{10} \quad \left[\Delta_D = \frac{384}{EI} (\downarrow) \right]$$

For 5kN.

$$\theta_A = \frac{t_{B/A}}{10} = \frac{128}{EI} (\curvearrowright)$$

$t_{A/B}$ = Moment of $\frac{M}{EI}$ diagram b/w A & B abt. A

$$= \frac{1}{2} \left(\frac{48}{EI} \right) \times 4 \times 2 \times \left(\frac{4}{3} \right) + \frac{1}{2} \times \left(\frac{48}{EI} \right) \times 6 \times \left(\frac{4+6}{3} \right)$$

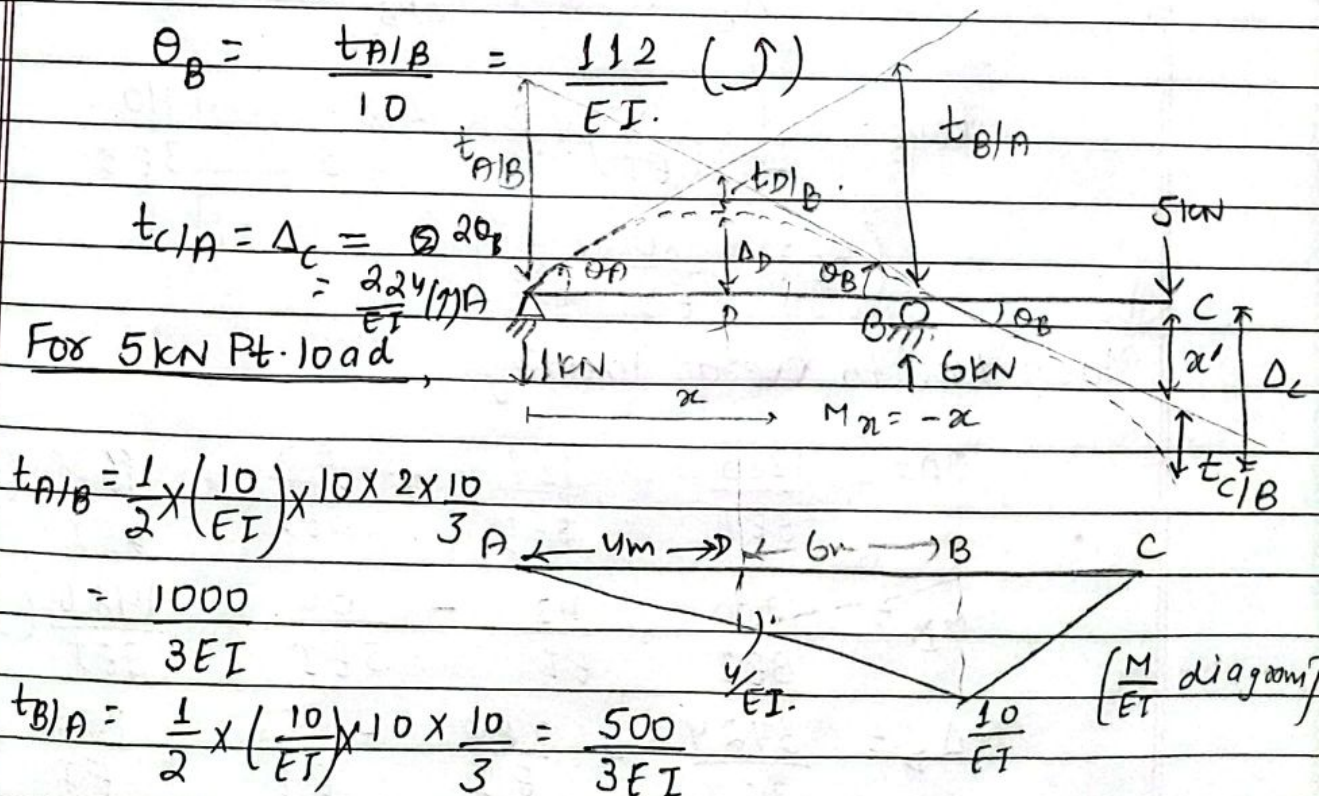
$$= \frac{1120}{EI}$$

$$\theta_B = \frac{t_{A/B}}{10} = \frac{112}{EI} (\curvearrowleft)$$

$$t_{C/A} = \Delta_C = \theta_B$$

$$= \frac{224}{EI} (\downarrow)$$

For 5kN Pt. load



$$t_{A/B} = \frac{1}{2} \times \left(\frac{10}{EI} \right) \times 10 \times 2 \times \frac{10}{3}$$

$$= \frac{1000}{3EI}$$

$$t_{B/A} = \frac{1}{2} \times \left(\frac{10}{EI} \right) \times 10 \times \frac{10}{3} = \frac{500}{3EI}$$

$$\theta_A = \frac{t_{B/A}}{10} = \frac{50}{3EI} (\uparrow)$$

$$\theta_B = \frac{t_{A/B}}{10} = \frac{100}{3EI} (\uparrow)$$

$$t_{D/B} = \frac{1}{2} \times \frac{4}{EI} \times 6 \times \frac{6}{3} + \frac{1}{2} \times \frac{10}{EI} \times 6 \times \frac{2 \times 6}{3}$$

$$= \frac{144}{EI}$$

$$\frac{\Delta_D + t_{D/B}}{t_{A/B}} = \frac{6}{10}$$

$$\Rightarrow \left[\Delta_D = \frac{456}{EI} \right] (\uparrow)$$

$$\theta_B = \frac{\alpha'}{2} \Rightarrow \alpha' = \frac{200}{3EI}$$

$$\Delta_C = \alpha' + t_{C/B}$$

$$t_{C/B} = \frac{1}{2} \times \left(\frac{10}{EI} \right) \times 2 \times \frac{2 \times 2}{3} = \frac{40}{3EI}$$

$$\left[\Delta_C = \frac{80}{EI} \right] (\downarrow)$$

Thus, due to overall loading,

$$\theta_A = \frac{1250}{3EI} + \frac{128}{3EI} - \frac{50}{3EI} = \frac{1328}{3EI} (\uparrow)$$

$$\theta_B = \frac{1250}{3EI} + \frac{112}{EI} - \frac{100}{3EI} = \frac{1486}{3EI} (\uparrow)$$

$$\Delta_D = \frac{3724}{3EI} + \frac{384}{EI} - \frac{56}{EI} = \frac{1569}{EI} (\downarrow)$$

$$\Delta_c = \frac{2500}{3EI} - \frac{80}{EI} + \frac{224}{EI} = \frac{2932}{3EI} \uparrow$$

Conjugate Beam Method.

An Imaginary Beam with $\frac{M}{EI}$ diagram of real beam as loading diagram is called conjugate beam.

(Real Beam)

$$\theta_A = \frac{t_{B/A}}{L}$$

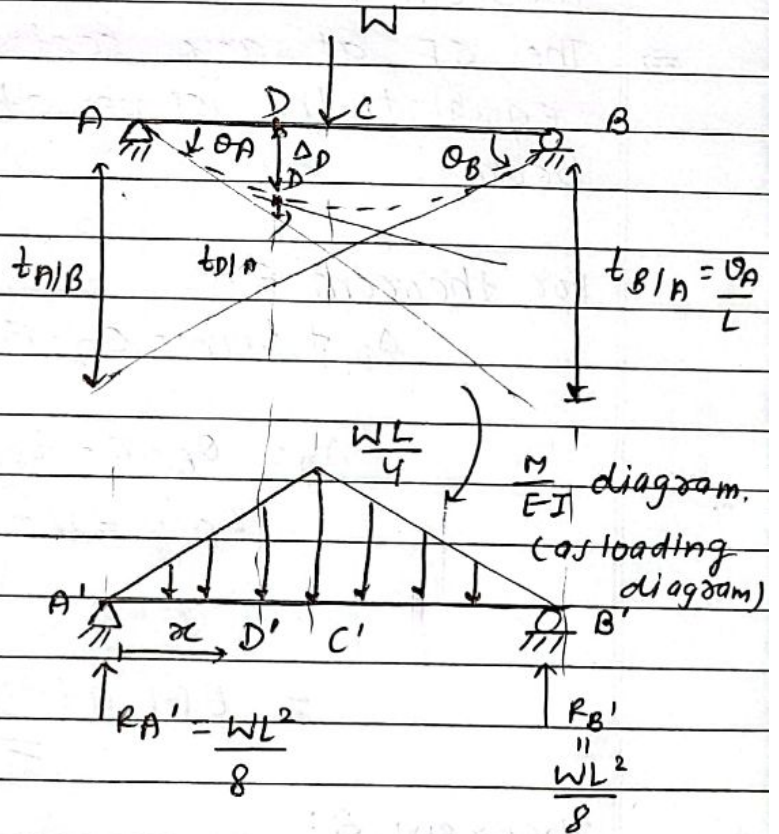
= Moment of $\frac{M}{EI}$ diagram between A and B abt B

$$= \frac{1}{2} \times \left(\frac{WL}{4}\right) \times L \times (L)$$

$$= \frac{WL^2}{8}$$

$$= R_{A'}$$

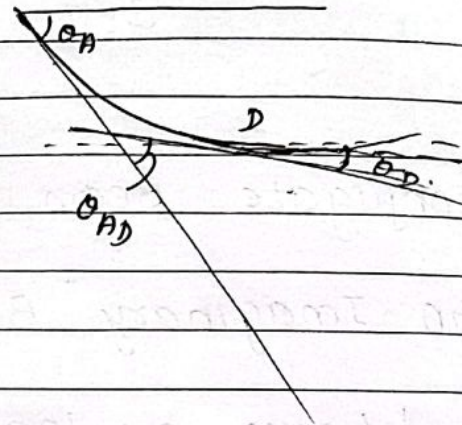
= SF at A' (Conj. beam)



$$\theta_D = \theta_A - \theta_{AD}$$

$$= R_A' y - \left(\text{Area of } \frac{M}{EI} \right. \\ \left. \text{dia. b/w A \& D} \right)$$

$$= \text{S.F. at D'}$$



Theorem 1:

⇒ The S.F at any section of conjugate beam is equal to the slope at that section of real beam.

For theorem 2:

$$\Delta_D + t_{D/A} = \theta_A \cdot x$$

$$\Delta_D = \theta_A \cdot x - t_{D/A}$$

$$= R_A' y \cdot x - \text{Moment of } \frac{M}{EI} \text{ dia b/w}$$

A and D abt. D.

$$= \text{BM at D'}$$

Theorem 2:

⇒ The BM at any section of conjugate beam is equal to the deflection at that section of Real beam.

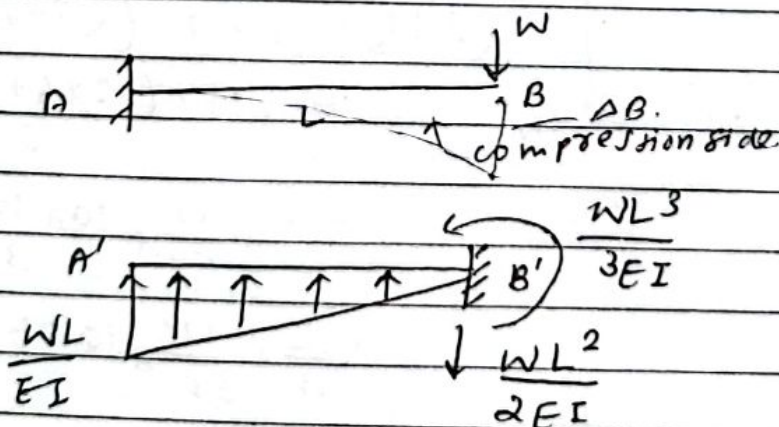
Rules for conjugate beam

- (1) $\frac{M}{EI}$ diagram $\rightarrow$ Loading
- (2) Free end $\rightarrow$ Fixed end
- (3) Fixed end $\rightarrow$ free end.
- (4) Interior roller support $\leftrightarrow$ Interior hinge
- (5) No change in roller and Hinge support at end.

Rules for Drawing B.M.D

- (1) Draw B.M.D on compression side.
- (2) Draw Arrow (loading) towards axis of beam.
- (3) +ve SF $\rightarrow$ clw slope
-ve SF $\rightarrow$ ACW slope
- (4) +ve B.M. $\rightarrow$ downward deflection.
-ve BM $\rightarrow$ upward deflection

eg:

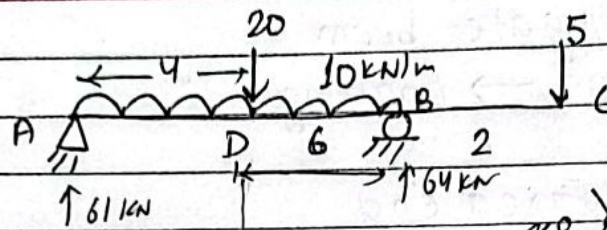


Thus, $\theta_B = \frac{WL^2}{2EI}$ (clockwise) = SF_{B'} (+ve)

$$\Delta_B = BM_{B'} = \frac{WL^3}{3EI} (\downarrow)$$

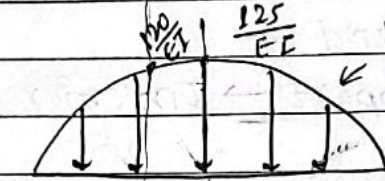
$\frac{1}{2} \times \frac{1}{3} \times \frac{1}{2} \times \frac{1}{2}$

Q.1

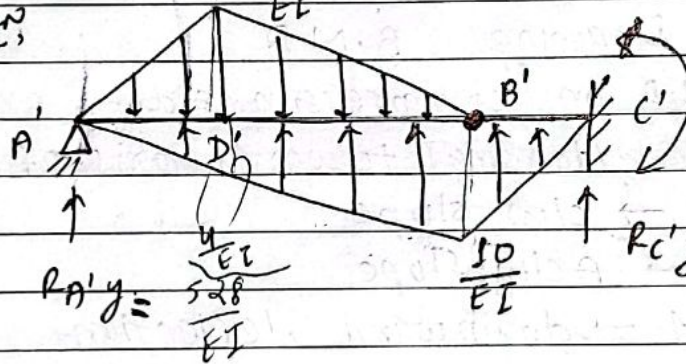


→ Soln:-

For UDL



For 20 kN



[Compression] ⇒ Diff.
Tension ⇒ attached to comp. side

$$M_C' = \frac{2932}{3EI}$$

$$R_C'y = \frac{1456}{3EI}$$

$$\begin{aligned} \theta_A &= SF_{A'} \\ &= R_{A'y} \\ &= \frac{528}{EI} \quad (2) \end{aligned}$$

$$\begin{aligned} \text{For } R_{A'y}, \\ (\sum M_{B'})_{\text{left}} &= 0 \quad (\curvearrowright +ve) \\ \text{or } \frac{1}{2} \times \frac{48}{EI} \times 10 \times \left(\frac{2 \times 6 + 4}{3} \right) \end{aligned}$$

$$- \frac{1}{2} \times \frac{10}{EI} \times 10 \times \frac{10}{3}$$

$$+ \frac{2}{3} \times \frac{125}{EI} \times 10 \times 5$$

$$- R_{A'y} \times 10 = 0$$

$$\Rightarrow R_{A'y} = \frac{528}{EI}$$

$$\theta_B = SF_B'$$

$$= R_A'y - \frac{2}{3} \times \frac{125}{EI} \times 10 - \frac{1}{2} \times \frac{48}{EI} \times 10 + \frac{1}{2} \times \frac{10}{EI} \times 10$$

$$= \frac{528}{EI} - \frac{193070}{3EI}$$

$$= \frac{-1986}{3EI}$$

$$= \frac{12486}{3EI} (\uparrow)$$

$$\Delta_D = BM_D'$$

$$= \left(\frac{528}{EI}\right) \times 4 - \int_0^4 \int_0^4 \frac{(10x^2 - 5x^2)(4-x)}{EI} dx - \frac{2}{3} \times \frac{120}{EI} \times 4 \times \frac{3 \times 4}{8}$$

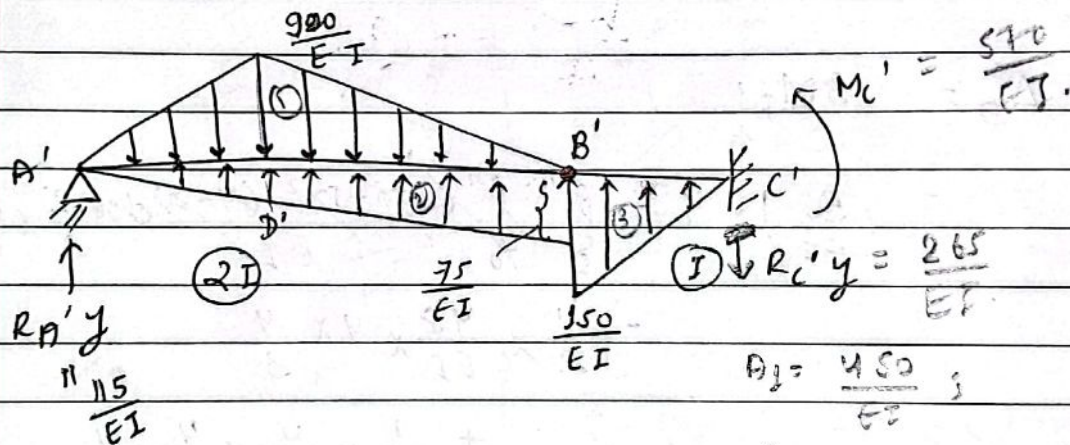
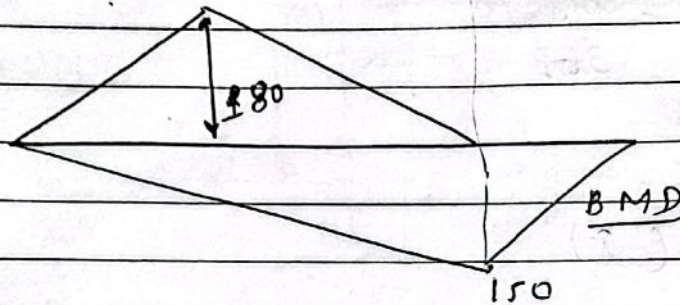
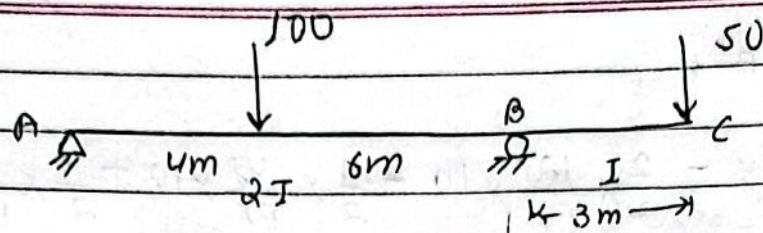
$$- \frac{1}{2} \times \frac{48}{EI} \times 4 \times \frac{4}{3} + \frac{1}{2} \times \frac{4}{EI} \times \frac{4}{3} \times 4$$

$$= \left(\frac{5960}{3} - 480\right) \frac{1}{EI} = \frac{1507}{EI}$$

$$= \left(2112 - 480 - 128 + \frac{32}{3}\right) \frac{1}{EI}$$

$$= \frac{1515}{EI} (\downarrow)$$

$$\Delta_C = BM_C' = \frac{2932}{3EI} (\uparrow)$$



$$(\sum M_{B'}) = 0 \quad (\uparrow +ve)$$

$$\text{or, } \frac{375}{EI} \times \frac{10}{3} - \frac{450}{EI} \times \frac{(2 \times 6 + 4)}{3} + R_{C'y} \times 10 = 0$$

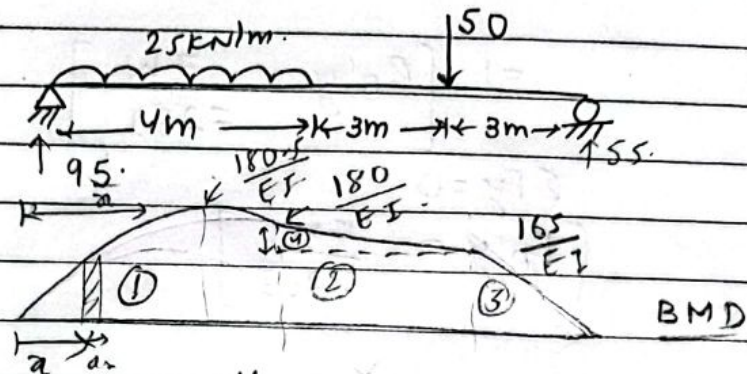
$$\Rightarrow R_{C'y} = \frac{115}{EI} \quad (\uparrow)$$

$$(\sum M_{C'})_{right} = 0 \quad (\uparrow +ve)$$

$$\text{or, } \frac{265}{EI} \times 3 - \frac{225}{EI} \times \frac{3}{3} - M_{C'} = 0$$

$$\Rightarrow \left[M_{C'} = \frac{570}{EI} \quad (\uparrow) \right]$$

119.

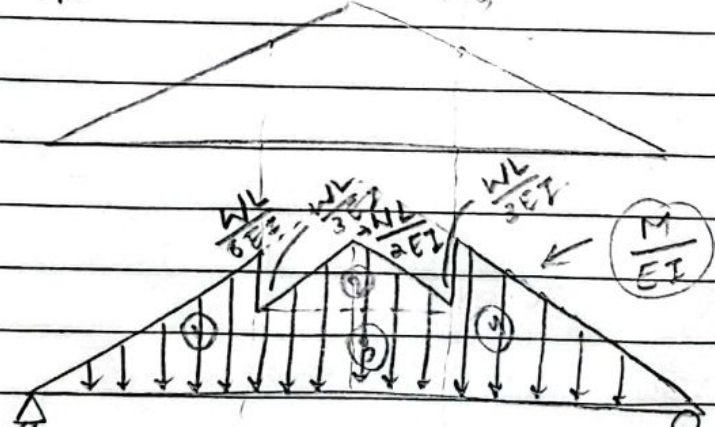
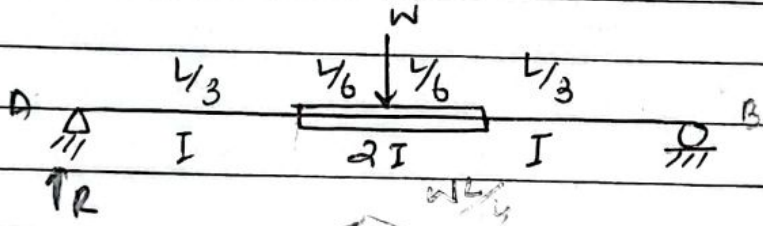


$$A_1 = \int_0^4 (95x - 12.5x^2) dx = 1480$$

$$\bar{x} = \frac{\int_0^4 x dx}{\int_0^4 1 dx} = \frac{3}{185}$$

$$(A) = \left(\frac{1480}{3} \right)$$

(9)



$\left(\frac{M}{EI} \right)$ dia as loading inth conjugate beam.

$$A_1 = \frac{WL}{36EI}, A_2 = \frac{WL}{36EI}, A_3 = \frac{WL}{9EI}, A_4 = \frac{WL}{36EI}$$

$$\Sigma M_{A'} = 0 \quad (\curvearrowright +ve) \quad \frac{WL}{36EI}$$

$$\text{or, } \underbrace{A_1 \times \frac{2}{3} \times \left(\frac{L}{3}\right)}_{\frac{WL}{36EI}} + A_2 \times \left(\frac{L}{3} + \frac{L}{6}\right) + A_3 \times \left(\frac{L}{3} + \frac{L}{6}\right) + A_4 \times \left(2 \times \frac{L}{3} + \frac{1}{3} \left(\frac{L}{3}\right)\right) - R_{B'y} \times L = 0$$

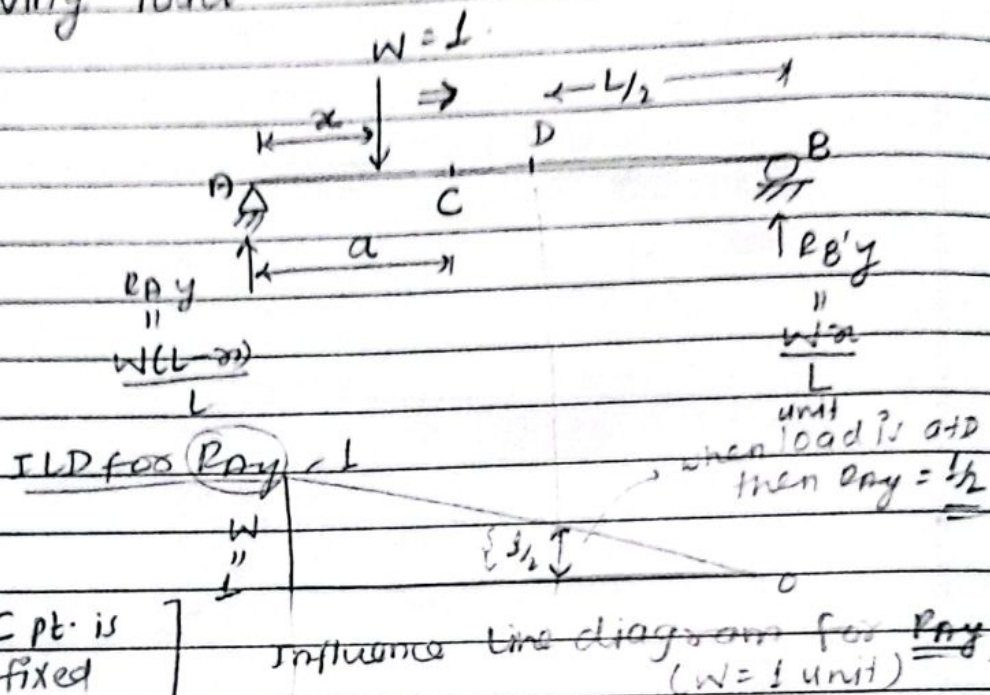
$$\Rightarrow \left[R_{B'y} = \frac{7W}{72EI} \right]$$

$$\Sigma F_y = 0$$

$$\left[R_{A'y} = \frac{7W}{72EI} \right]$$

⑤ Influence Line Diagram (ILD)

→ Moving load



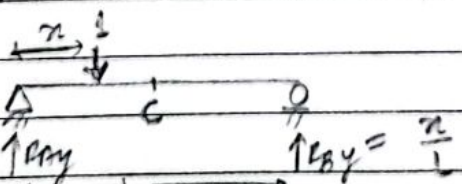
C pt. is fixed
↓
Two function
→ SF
→ BM

Influence line diagram for R_{Ay}
($W=1$ unit)

ILD for R_{By}

• SF at c

When unit load is in portion AC ($0 \leq x \leq a$)

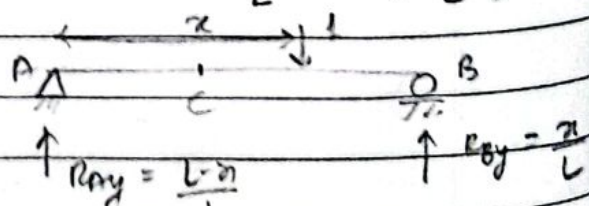
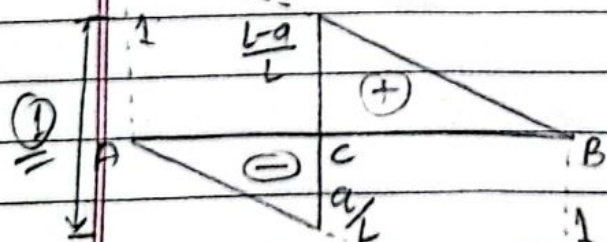


$$SF_c = -R_{By} = -\frac{x}{L}$$

$$x=a, SF_c = -\frac{a}{L}, x=0, SF_c = 0$$

When unit load is in portion CB ($a \leq x \leq L$)

$$SF_c = R_{Ay} = \frac{L-x}{L} = \frac{L-a}{L}$$



$$x=a, SF_c = \frac{L-a}{L}$$

$$x=L, SF_c = 0$$

• BM at C (M_c)

→ When unit load is in portion AC ($0 \leq x \leq a$)

$$BM_c = R_{By} \times (L - a) = \frac{x(L - a)}{L}$$

$$\text{when, } x = 0, M_c = 0$$

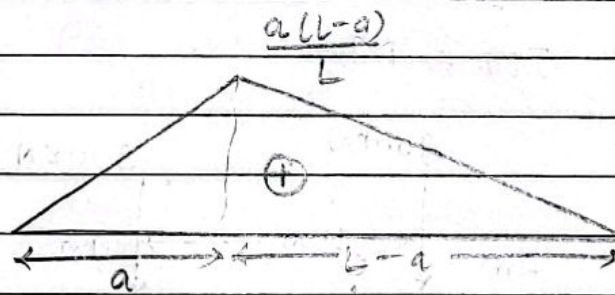
$$\text{when, } x = a, M_c = \frac{a(L - a)}{L}$$

→ When unit load is in portion CB ($a \leq x \leq L$)

$$BM_c = R_{Ay} \times a = \frac{a(L - x)}{L}$$

$$\text{when, } x = a, BM_c = \frac{a(L - a)}{L}$$

$$\text{when, } x = L, M_c = 0$$



ILD for BM_c

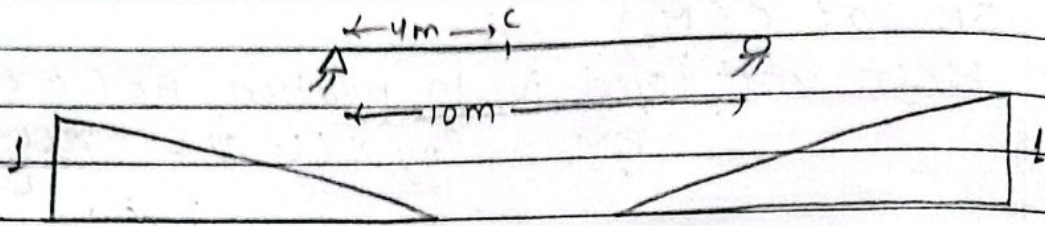
[+ve → मावि]

[-ve → तन]

• Influence Line Diagram in which

It is a diagram, the ordinate at any point represents the value of that function when unit load is acting at that point

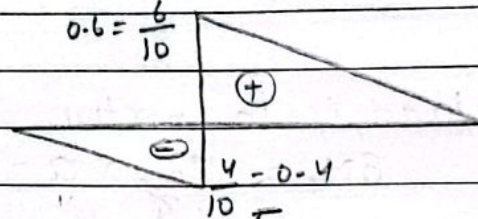
g. Draw ILD for for R_A , R_B , SF & M_C .



ILD for R_A

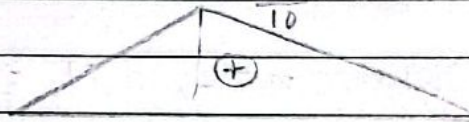
ILD for R_B .

$$0.6 = \frac{6}{10}$$



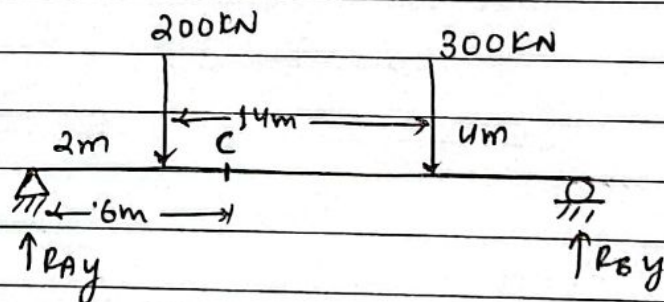
ILD for F_c

$$\frac{4 \times 6}{10} = 2.4$$



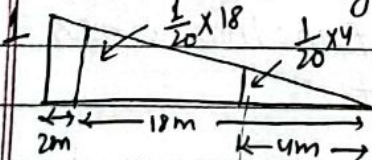
ILD for M_C

Use of ILD

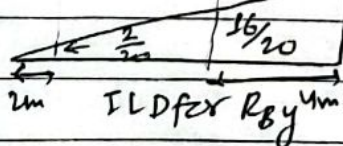


Using ILD find R_{Ay} , R_{By} , F_c & M_C .

For 200kN loading



ILD for R_{Ay}



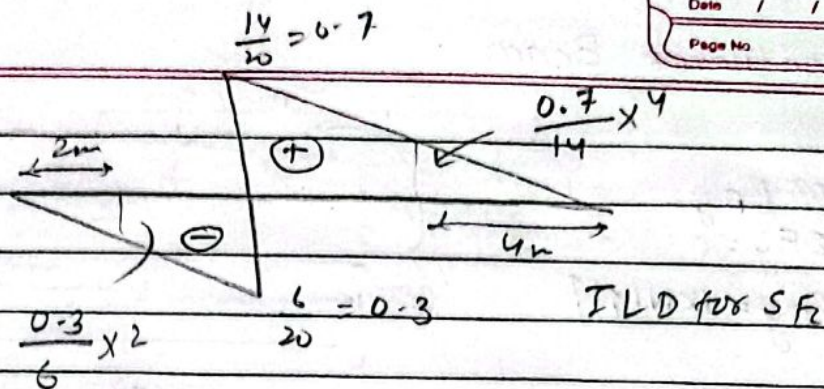
ILD for R_{By}

$$R_{Ay} = 200 \times \frac{18}{20} + 300 \times \frac{4}{20}$$

$$= 240 \text{ kN}(\uparrow)$$

$$R_{By} = 200 \times \frac{2}{20} + 300 \times \frac{16}{20}$$

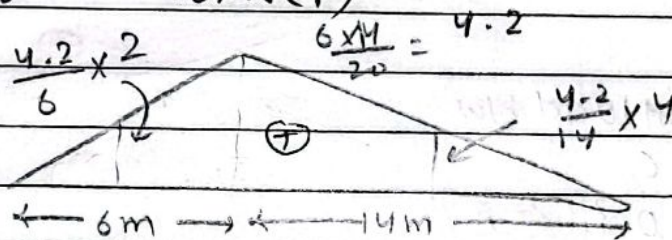
$$= 260 \text{ kN}(\uparrow)$$



For given loading,

$$SF_2 = \left(-\frac{0.3}{6} \times 2 \times 200 \right) + \left(\frac{0.7}{14} \times 4 \right) \times 300$$

$$SF_2 = 40 \text{ kN} (\uparrow)$$



For given loading,

$$BM_c = \left(\frac{4.2}{6} \times 2 \times 200 \right) + \left(\frac{4.2}{14} \times 4 \right) \times 300$$

For
unit
load

$$= 640 \text{ kNm} (\rightarrow)$$

For unit
load

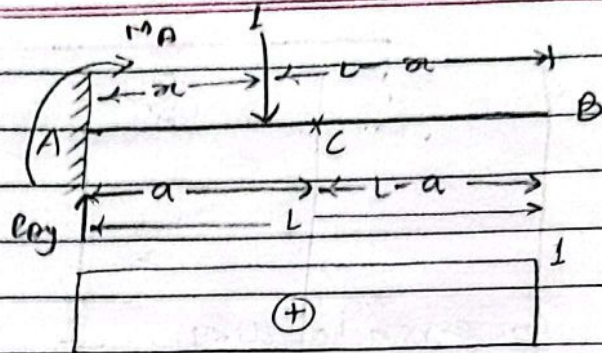
*Max^m BM at C will occur when Load is at C.

* Cantilever Beam

• For R_{Ay}

$$\sum F_y = 0$$

$$[R_{Ay} = 1 \text{ KN}(\uparrow)]$$



• For M_A

$$[\sum M_A = 0]$$

$$M_A = -1 \times x$$

$$x=0, M_A=0; x=L, M_A=-L$$

ILD for R_{Ay}

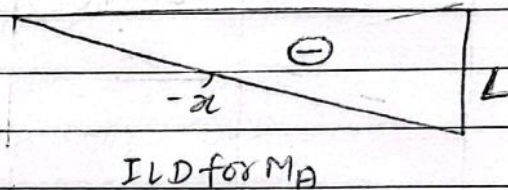
• For SF_C

When unit load b/w
A and C,

$$SF_C = 0 \quad \{L < x\}$$

When unit load b/w
C and B

$$SF_C = 1 \quad \{L < x\}$$



ILD for M_A

ILD for SF_C

• For M_C

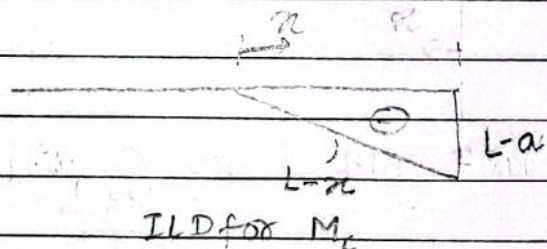
When unit load b/w
A and C,

$$M_C = 0 \quad [L < x]$$

When unit load b/w

C and B ($0 \leq x \leq a$)

$$M_C = -[x(L-x)]$$

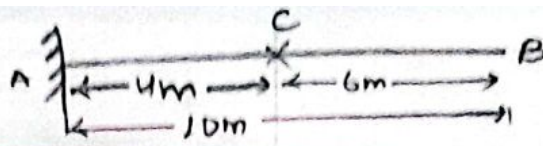


ILD for M_C

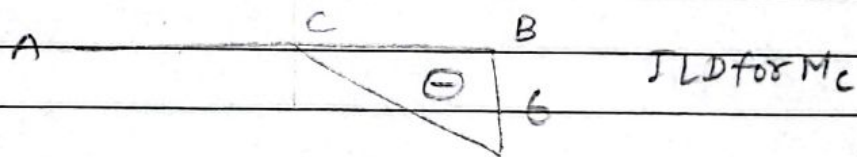
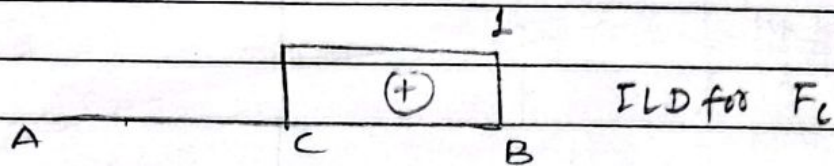
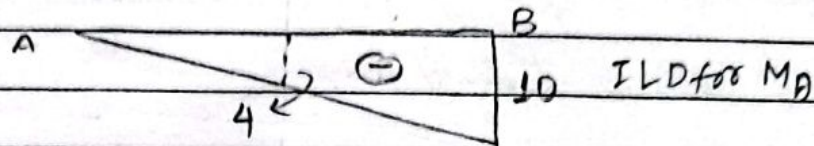
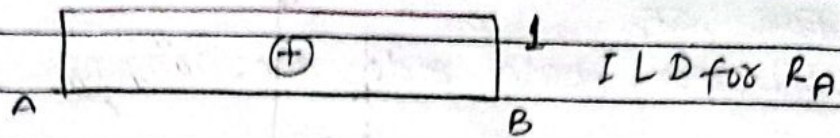
[If x from A, $a \leq x \leq L$]

$$M_C = -(x-a)$$

Q.



Date / /
Page No.

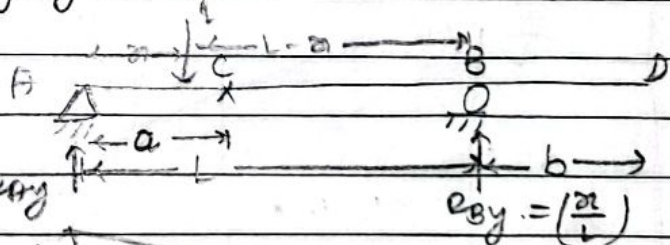


* ILD for overhanging beam.

For R_{Ay}

$$\sum M_B = 0 \quad [\sum M_B = 0]$$

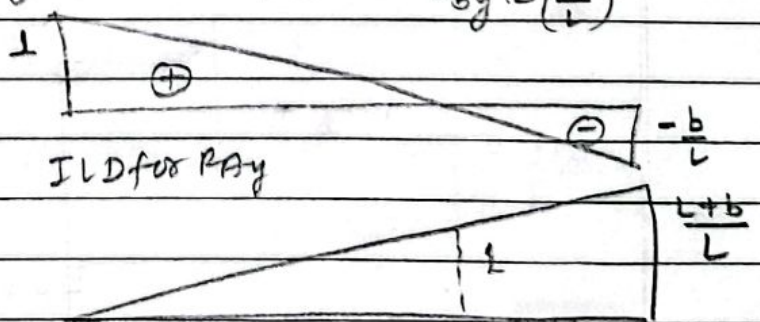
For AB: $R_{Ay} = \left(\frac{L-a}{L}\right) \left(\frac{L-a}{L}\right) = R_{Ay}$



For BC: $R_{Ay} \times L + 1 \times (a-L) = 0$

$$\Rightarrow R_{Ay} = -\left(\frac{a-L}{L}\right)$$

$$[L \leq a \leq L+b]$$



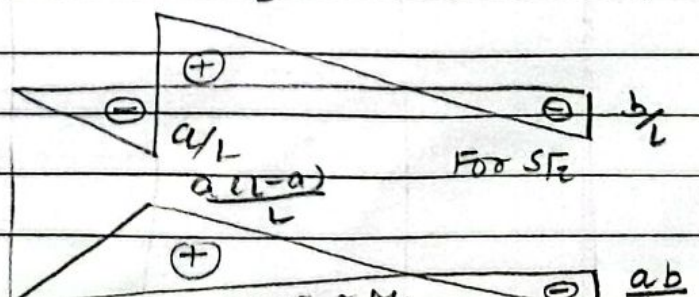
$$\left(\frac{L-a}{L}\right)$$

ILD for R_{By}

For R_{By} ,

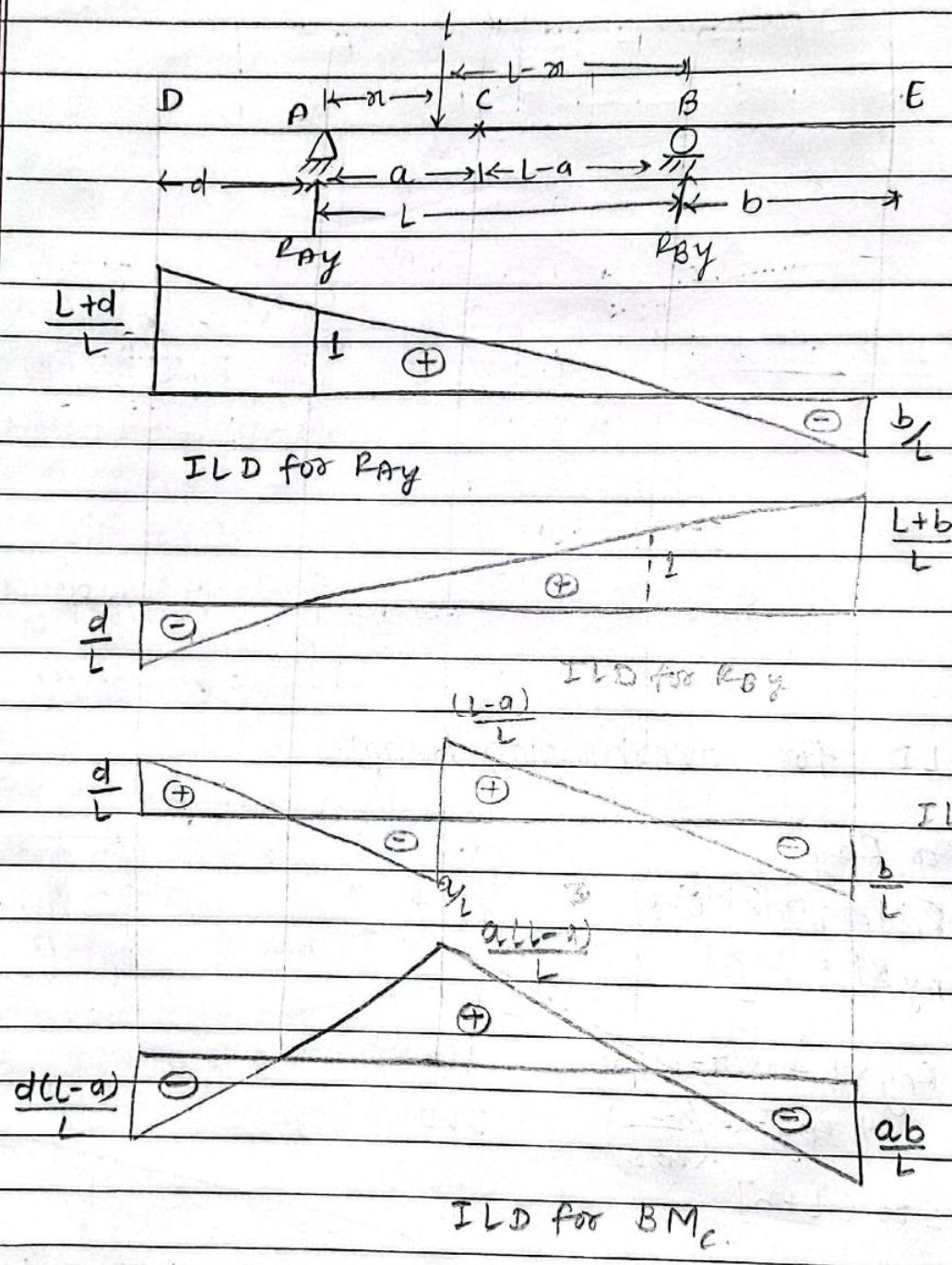
$$\sum M_A = 0$$

$L \leq a \leq L+b$ $[R_{By} = \frac{a}{L}]$

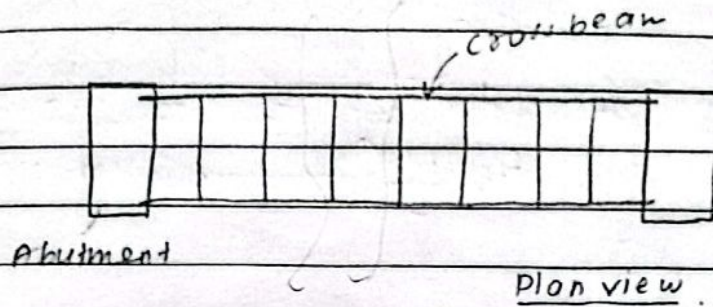


For SF_C

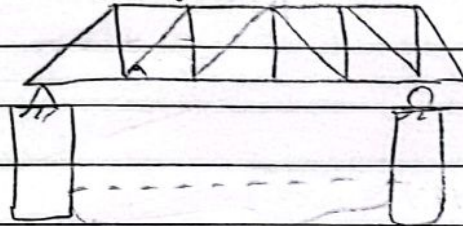
- For SF_c
- * ILD for both side overhanging beam.



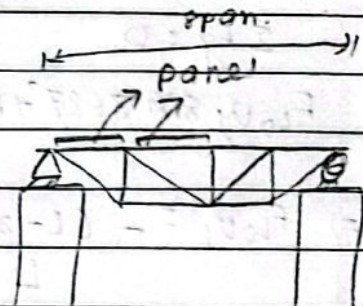
ILD for member force in a truss.



Thorough type → तारु माधि



Thorough type

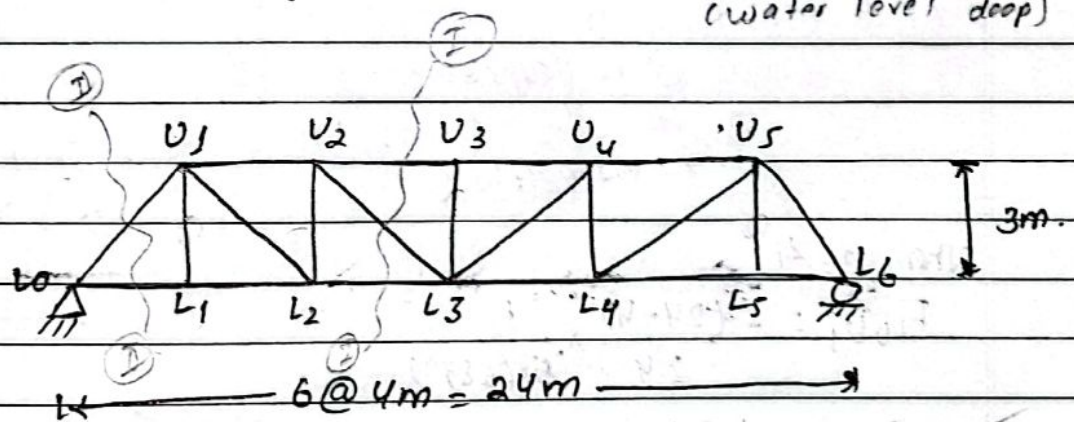


Panel → distance between two joints

Deck type

(water level deep)

(Q)



Draw ILD for forces in members $L_0U_1, L_0L_1, U_1L_1, U_2U_3, U_2L_3, L_2L_3, U_3L_3$.

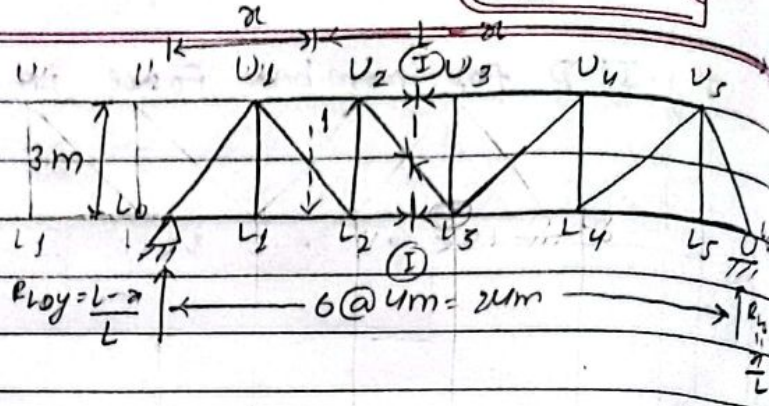
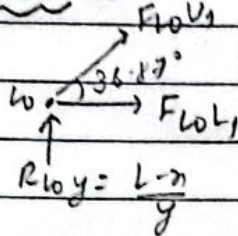
zero force memb.

Tension

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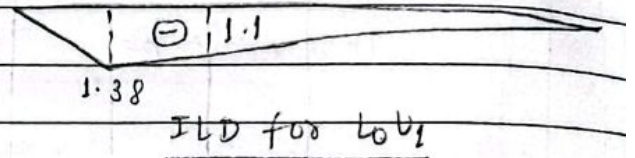
For $L_0 U_1$,

Joint L_0



$$\sum F_y = 0$$

$$\text{or } F_{L0 U1} \sin 36.87^\circ + R_{L0 y} = 0$$

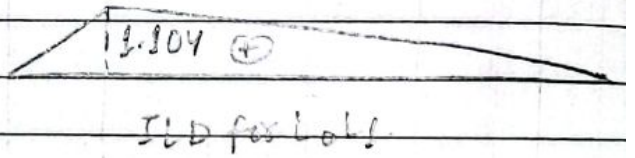


$$\Rightarrow F_{L0 U1} = - \frac{(L-a)}{L} \times \frac{1}{\sin(36.87^\circ)}$$

When $a=0$,

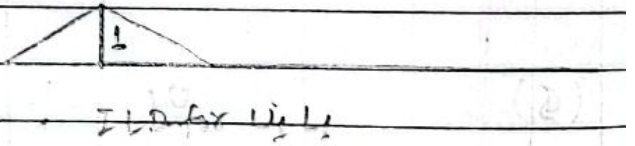
$$F_{L0 U1} = - \frac{1}{\sin(36.87^\circ)}$$

But don't consider it



At $a=0 \rightarrow F_{L0 U1} = 0$

(Load directly support
at joint doesn't
goes to member $L_0 U_1$)



U1 L1 zero force member
But due to moving
load directly above L1

When $a=4$,

$$F_{L0 U1} = - \frac{(24-4)}{24} \times \frac{1}{\sin 36.87^\circ}$$

$$= -1.38$$

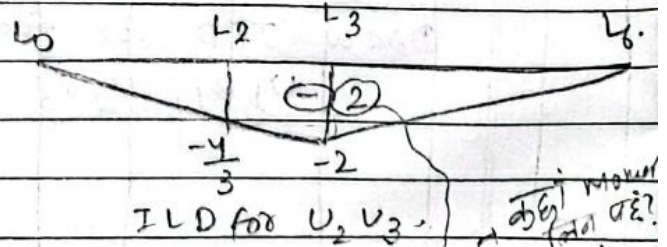
For $L_0 L_1$,

$$\sum F_x = 0$$

$$\text{or } F_{L0 U1} \cos(36.87^\circ) + F_{L0 L1} = 0$$

$$F_{L0 L1} = (- F_{L0 U1} \cos(36.87^\circ))$$

$$F_{L0 L1} = \frac{(L-a)}{L} \cot(36.87^\circ)$$



Moment $\frac{a(L-a)}{L \times h}$

$$= \frac{12 \times 12}{24 \times 3}$$

$$= 2$$

only for panel

For $U_2 U_3$, $U_2 L_3$, $L_2 L_3 \rightarrow$ consider section ①-②

For $U_2 U_3$

$\rightarrow$ cut force exists and force is zero moment
दिए ठी choose जहाँ (can be anywhere)

When unit load is on left of section ①-②
consider equilibrium of right part,

$$\sum M_{L_3} = 0 \quad (\curvearrowright +ve)$$

$$\text{or, } -F_{U_2 U_3} \times 3 - \left(\frac{x}{L}\right) \times 12 = 0$$

$$\Rightarrow F_{U_2 U_3} = -\frac{4x}{24} = -\frac{x}{6}$$

$$\text{For, } x=0, F_{U_2 U_3} = 0$$

$$\text{For } x=L_2, x=8, F_{U_2 U_3} = -\frac{4}{3}$$

When unit load is on right of section ①-②
consider equilibrium of left part;

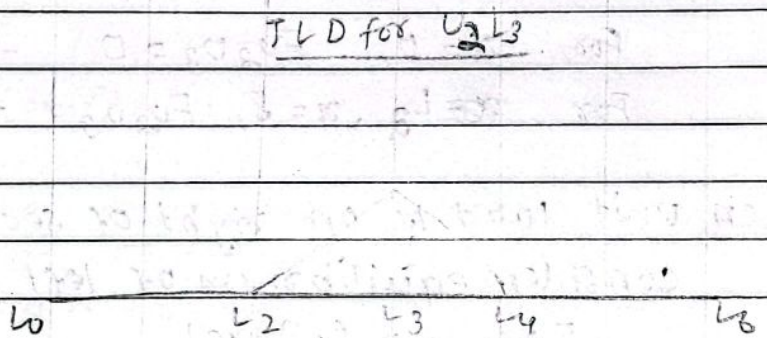
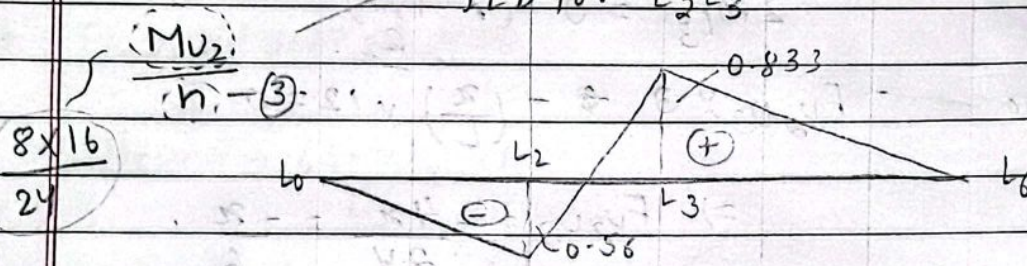
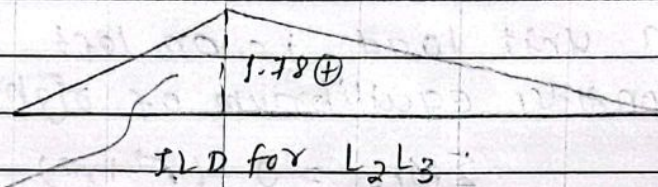
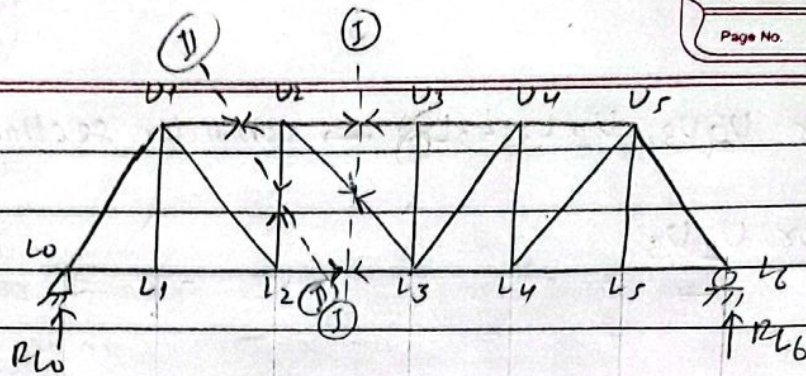
$$\sum M_{L_3} = 0 \quad (\curvearrowright +ve)$$

$$\text{or, } F_{U_2 U_3} \times 3 + \left(\frac{L-x}{L}\right) \times 12 = 0$$

$$\Rightarrow F_{U_2 U_3} = -\frac{4(24-x)}{24} = -\frac{(24-x)}{6}$$

$$\text{For, } x=12, L_3, F_{U_2 U_3} = -2$$

$$x=24, L_6, F_{U_2 U_3} = 0.$$

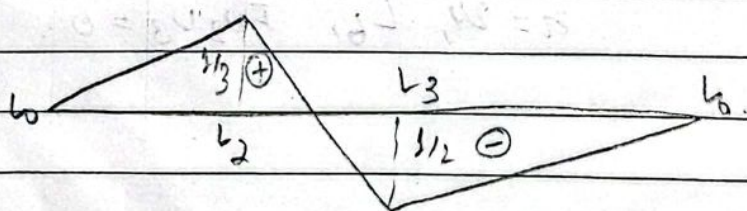


ILD for U_3L_3

(L_3 is load on inclined member at joint)

If vertical member at load joint

U_3 is U_2 balance



For $F_{U_2L_4}$

For $L_2 L_3$,

When unit load is left of L_2 ,

$$(L_0 - L_2) (\sum M_{U_2})_{\text{right}} = 0$$

$$\text{or, } F_{L_2 L_3} = \frac{R_{L_6} \times 16}{3} (T)$$

$$\text{For, } L_0, n = 0, F_{L_2 L_3} = 0$$

$$n = 8, F_{L_2 L_3} = 1.78 (T)$$

When unit load is right of L_2 ,

$$(3 - L_6) (\sum M_{U_2})_{\text{left}} = 0$$

$$\text{or, } F_{L_2 L_3} = \frac{R_{L_6} \times 8}{3} (T)$$

$$\text{when, } n = 12, F_{L_2 L_3} = 1.33 (T)$$

$$n = 24, F_{L_2 L_3} = 0$$

For $U_2 L_3$,

When, unit load left of L_2 ,

$$(\sum F_y)_{\text{right}} = 0$$

$$F_{U_2 L_3} = \frac{R_{L_6}}{\sin(36.867^\circ)} (C)$$

$$L_0 = 0 \quad L_2 = 0.56$$

When, unit load right of L_3 ,

$$(\sum F_y)_{\text{left}} = 0$$

$$\text{or, } F_{U_2 L_3} = \frac{R_{L_0}}{\sin(36.867^\circ)} (T)$$

$$L_3 = 0.833 \quad L_6 = 0$$

For $U_2 L_2$

taking section (I)-(I)

For unit load b/w L_0 & L_2

$$(\sum F_y)_{right} = 0$$

$$\text{or } F_{U_2 L_2} = R_{L_6} = \frac{x}{L} (+)$$

$$x = 0(L_0)$$

$$0$$

$$x = 8(L_2)$$

$$\frac{8}{24} = \frac{1}{3}$$

For unit load b/w L_3 & L_6

$$(\sum F_y)_{left} = 0$$

$$F_{U_2 L_2} = R_{L_0} = \frac{L-x}{L} (-)$$

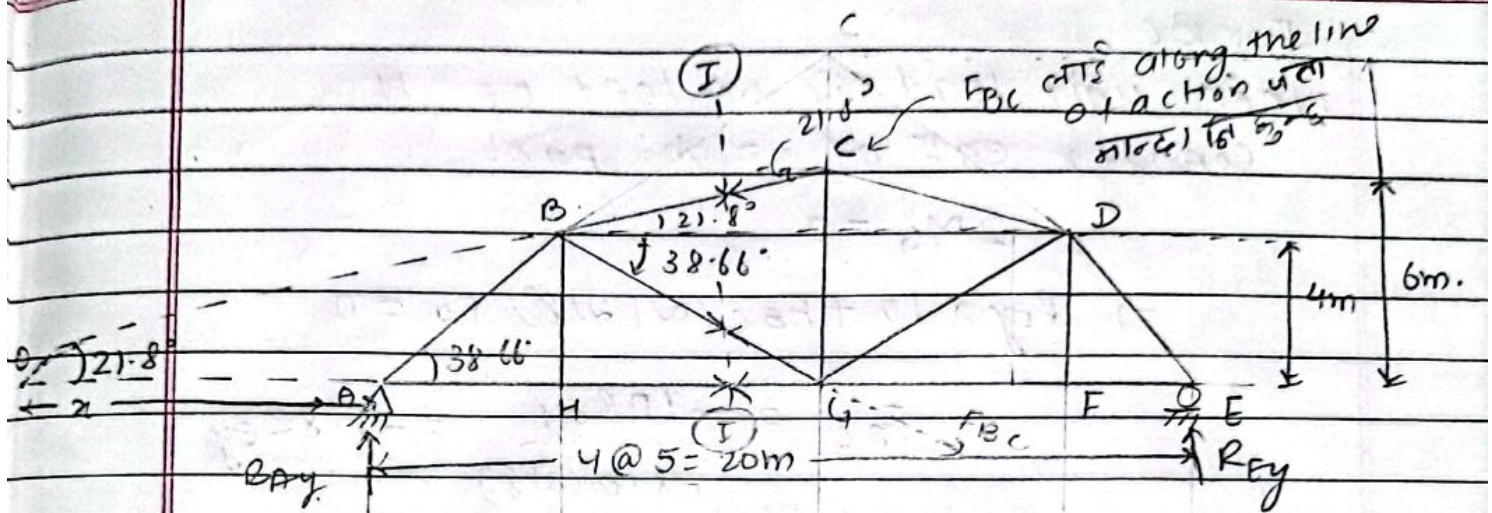
$$L_3$$

$$x = 12$$

$$\frac{12}{24} = \frac{1}{2}$$

$$L_6$$

$$0.$$



From similar

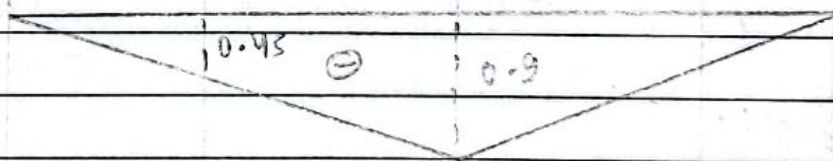
BC, BG, GH

ΔDBH

ΔCG

$[n=5]$

$[1=5]$



For BC

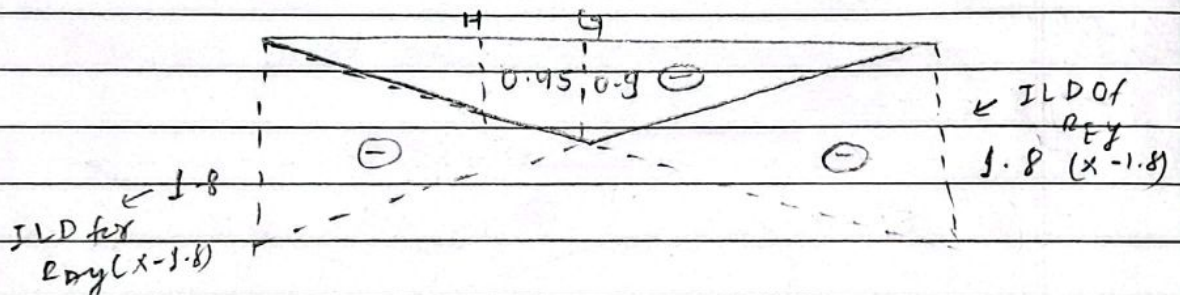
When unit load is on left of H

Consider eq^m of right part.

$$\sum M_G = 0$$

$$\Rightarrow R_{EY} \times 10 + F_{BC} \cos(21.8^\circ) \times 6 = 0$$

$$F_{BC} = \frac{-10 R_{EY}}{6 \cos(21.8^\circ)} = -1.8 R_{EY}$$



When unit load is on right of G

consider left part in eq^m

$$\sum M_G = 0$$

$$\text{or, } R_{EY} \times 10 + F_{BC} \sin 21.8^\circ \times 15 = 0$$

$$F_{BC} = (-1.8) R_{EY}$$

Consider F_{BC} passing through G

only vertical compo for M_G

For HG

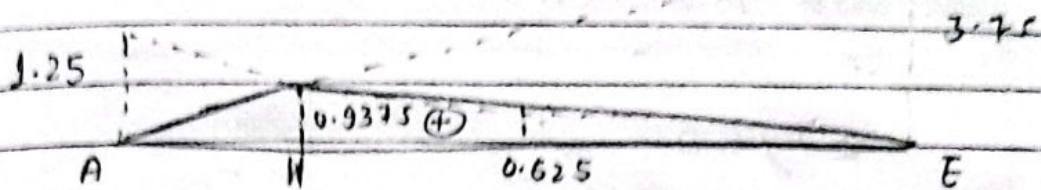
When unit load is on left of H,

consider eq^m of right part.

$$\sum M_B = 0$$

$$\text{or, } -R_{EY} \times 15 + F_{GH} \times 4 = 0$$

$$[F_{GH} = (43.75) R_{EY}]$$



When unit load is on right of G,
consider eq^m of left part

$$\Sigma M_B = 0$$

$$\text{or, } R_{Ay} \times 1.5 - F_{GH} \times 4 = 0$$

$$F_{GH} = 1.25(R_{Ay})$$

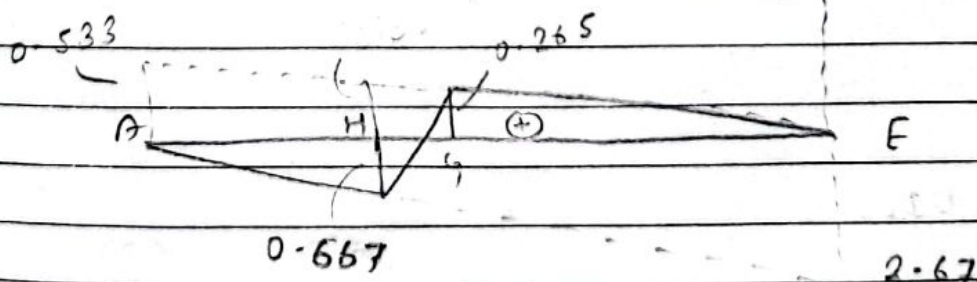
For BG,

When unit load is on left of H. Consider eq^m
of right part.

$$\Sigma M_D = 0$$

$$\text{or, } R_{Ey} \times 2.5 + F_{BG} \times \sin(38.66^\circ) = 0$$

$$F_{BG} = (-2.67)R_{Ey}$$



When, unit load is on right of H. consider eq^m
of left part = 0

$$\Sigma M_D = 0$$

(considering F_{BG} passing

$$\text{or, } F_{BG} = \frac{R_{Ay} \times 5}{\sin(38.66^\circ) \times 15} = 0.533 R_{Ay}$$

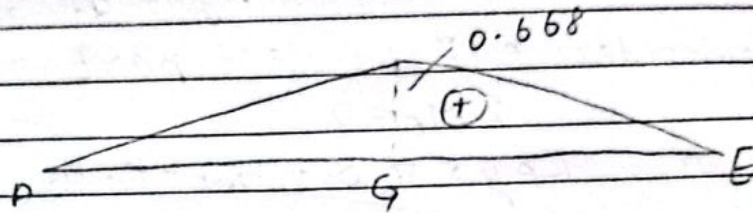
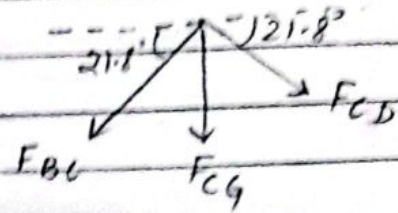
For CG, Use Joint Method,

$$F_{BC} = F_{CD}$$

$$\sum F_y = 0$$

$$F_{CG} + F_{BC} \sin 21.8^\circ = 0$$

$$F_{CG} = (-\sin 21.8^\circ) F_{BC} = (-0.371) F_{BC}$$

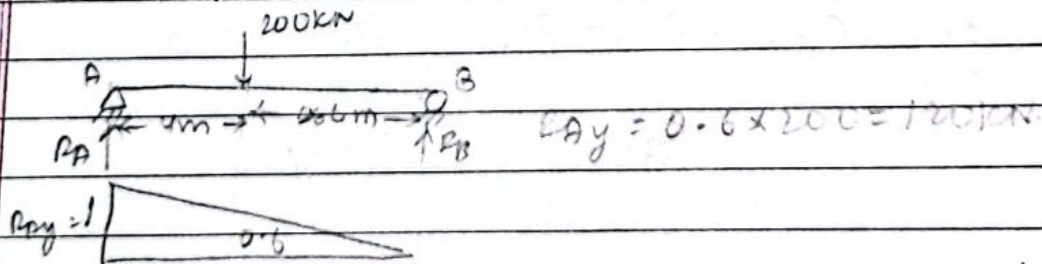


Use of ILD:

(1) For point load,

value of function = ordinate of ILD $\times$ Magnitude of load.

function $\rightarrow$ Reaction; shear force; Moment



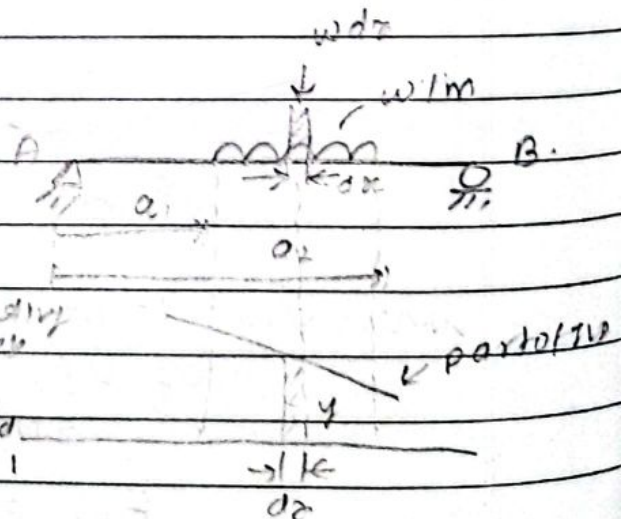
(2) For UDL,

Value of function

$$= \int_{a_1}^{a_2} (w dx) y \rightarrow \text{corresponding ordinate}$$

for small strip dx consider pt load

$$= w \int_{a_1}^{a_2} y dx$$



Thus,

For UDL,

Value of function = Magnitude of UDL
* Area of ILD under the UDL

(3) For couple (2):

Value of function = Magnitude of Couple *
Slope of ILD

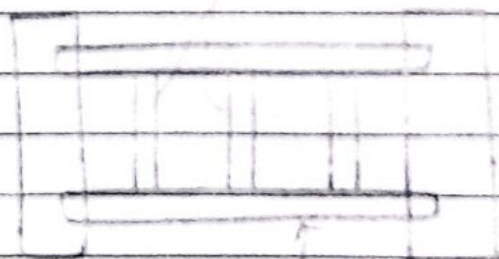
$$R_{dy} = (-1) \times M$$

(Slope clockwise)



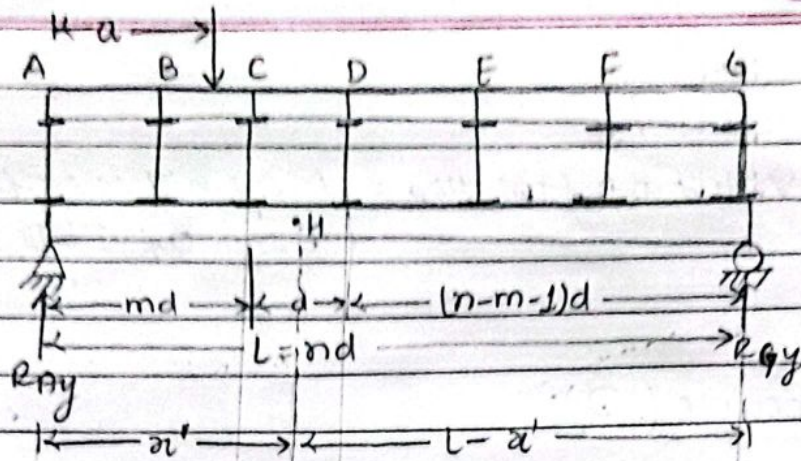
ILD for Key

* ILD for Girder

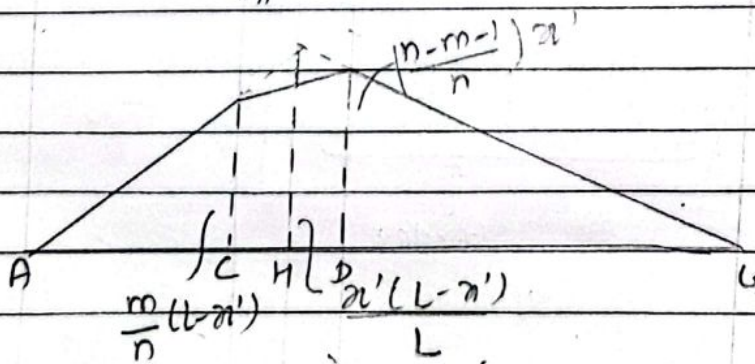
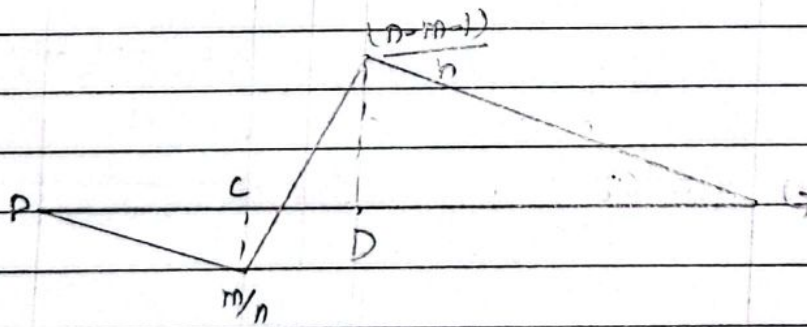
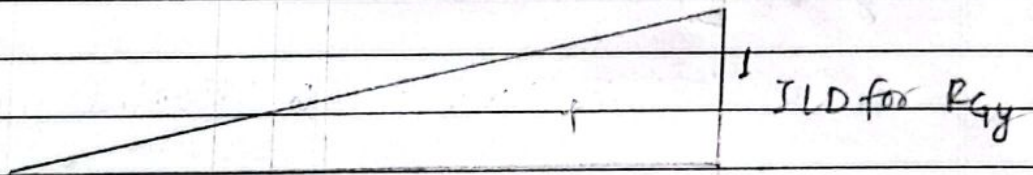
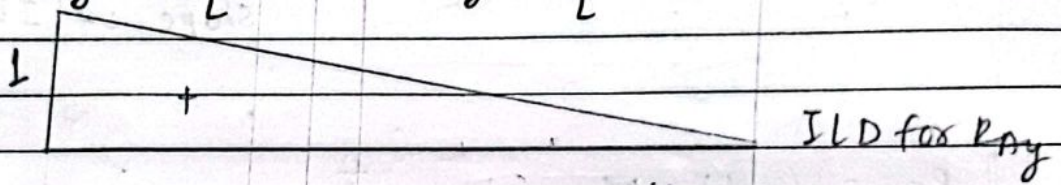


Girder

Plan



$$R_{Ay} = \frac{L - a}{L}, \quad R_{Gy} = \frac{a}{L}$$



similar to
simply
supported
beam

• SF for panel CD

(i) When unit load is on left of C

$$SF \text{ at panel } CD = -R_{Ay} \\ = -\frac{a}{L}$$

When unit load is at A, $a=0$, $SF_{CD}=0$

$$\text{" " " " C, } a=md, SF_{CD} = \frac{-md}{nd} = -\frac{m}{n}$$

(ii) When unit load is on right of D

$$SF \text{ at panel } CD = R_{Ay} = \frac{L-a}{L}$$

When unit force at D, $a=(m+1)d$

$$SF_{CD} = \frac{nd - (m+1)d}{nd} = \frac{n-m-1}{n}$$

When unit force at G, $a=nd$

$$SF_{CD} = \frac{nd - nd}{nd} = 0.$$

• For BM at h

i- When unit load is on left of C

$$M_h = R_{Ay} \times (L-a') \\ = \frac{a}{L} \times (L-a')$$

When, $a=0$, $M_h = 0$

$$a=md, M_h = \frac{md}{nd} (L-a') = \frac{m}{n} (L-a')$$

(ii) When unit load is on right of D

$$M_h = R_{Ay} \times a' = \left(\frac{L-a}{L} \right) a'$$

$$\text{When, } a=(m+1)d, M_h = \frac{nd - (m+1)d}{nd} a'$$

$$= \frac{n-m-1}{n} a'$$

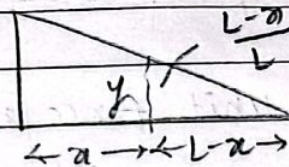
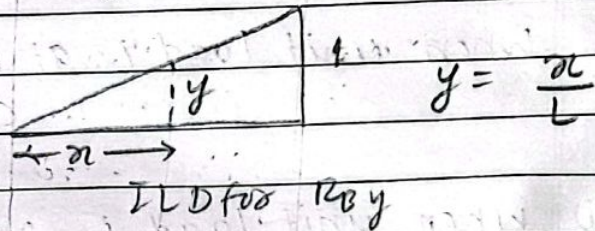
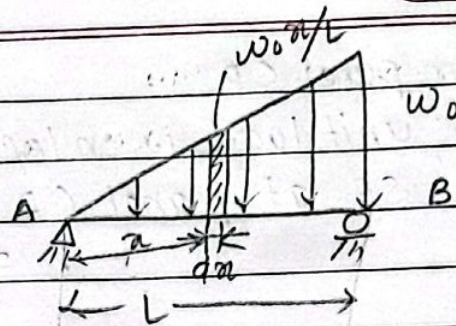
when, $a=nd$, $M_h = 0.$

For UVL,

$$R_{By} = \int \left(\frac{w_0 x}{L} dx \right) y$$

$$= \int_0^L \frac{w_0 x}{L} dx \cdot \frac{x}{L}$$

$$= \frac{w_0 L}{3}$$



$$R_{Ay} = \int_0^L \left(\frac{w_0 x}{L} \right) dx \times \left(\frac{L-x}{L} \right)$$

$$= \frac{w_0}{L^2} \int_0^L \left[\frac{Lx^2}{2} - \frac{x^3}{3} \right] dx$$

$$= \frac{w_0}{L^2} \left[\frac{L^3}{2} - \frac{L^3}{3} \right]$$

$$= \frac{w_0 L}{6}$$

Use of I.L.D. :- # Moving Load

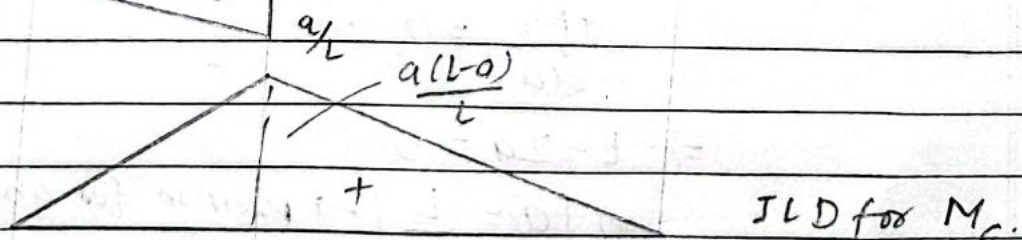
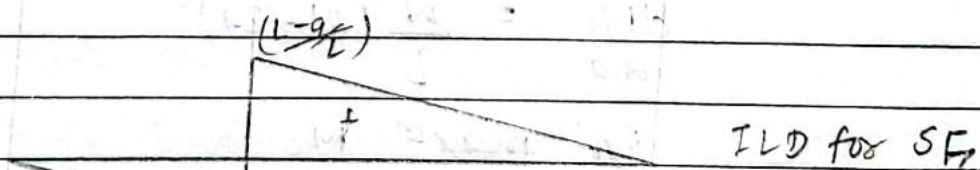
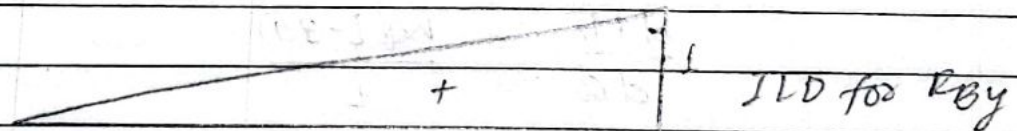
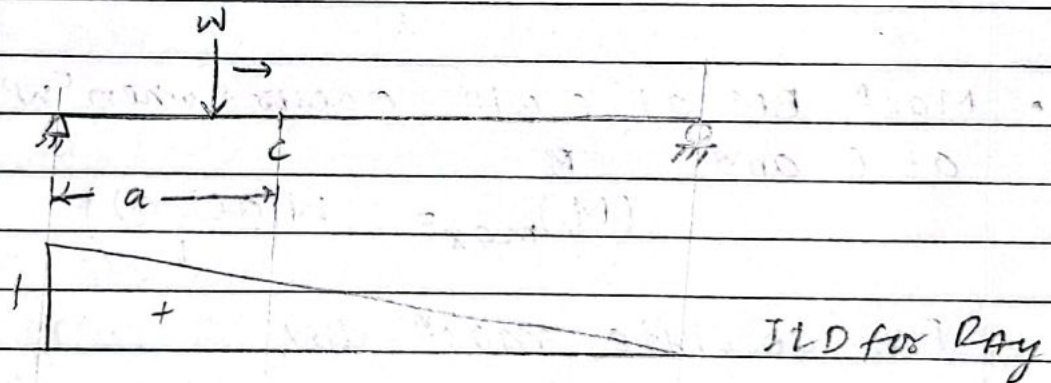
- (1) Single point load
- (2) UDL shorter than span
- (3) UDL longer than span
- (4) Several concentrated loads $\rightarrow$ Practical (vehicle)

Finding

- (i) Max^m S.F. at any section
- (ii) Max^m B.M at any section
- (iii) Absolute maximum S.F.
- (iv) Absolute maximum B.M.

section/pt. given
Don't mentioned section

(1) Single point load:



- Max^m Negative S.F. at C will occur when 'W' is acting at just left of 'C'
And it is

$$\text{Max}^{\text{ve}} \text{ S.F.}_c = \frac{W a}{L}$$

- Max^{ve} ~~negative~~ S.F. at C will occur when 'W' is acting at just right of 'C'

$$\text{Max}^{\text{ve}} \text{ S.F.}_c = \frac{W(L-a)}{L}$$

- Absolute max^{ve} S.F. when 'W' at A, $\frac{W}{L}$
- Absolute max^{ve} -ve S.F. when 'W' at B, $\frac{W}{L}$

- Max^{ve} BM at C will occur when 'W' is acting at C and B

$$(M_c)_{\text{max}} = \frac{W(a(L-a))}{L}$$

- For absolute max^{ve} BM, (C moving along with load)

$$\frac{dM_c}{da} = \frac{W(L-2a)}{L}$$

$$\frac{dM_c}{da} = \frac{W}{L}(L-2a)$$

$$\text{For, max}^{\text{ve}} M_c$$

$$\frac{dM_c}{da} = 0$$

$$\Rightarrow L - 2a = 0$$

$$\Rightarrow \left[a = \frac{L}{2} \right] \rightarrow \text{position for abs. max}^{\text{ve}}$$

$$(M_{\text{max}})_{\text{absolute}} = \frac{W \left(\frac{L}{2} \right) \left(L - \frac{L}{2} \right)}{L} = \frac{WL}{4}$$

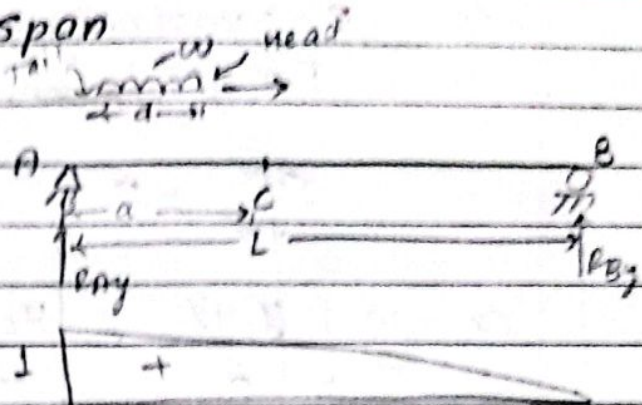
UDL shorter than span
 $[d < L]$

Max^m (R_p)

↳ when tail at A

Max^m R_B

↳ when head at B

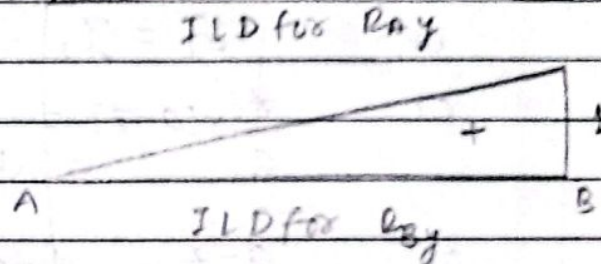


Max^m -ve SF_C,

↳ when head at C

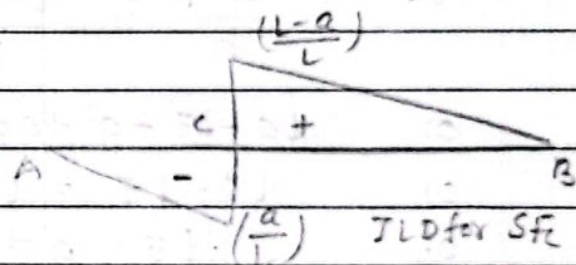
Max^m +ve SF₂

↳ when Tail at C

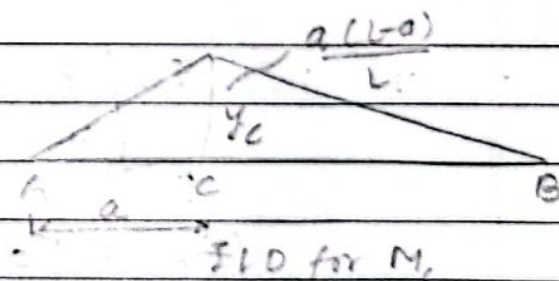
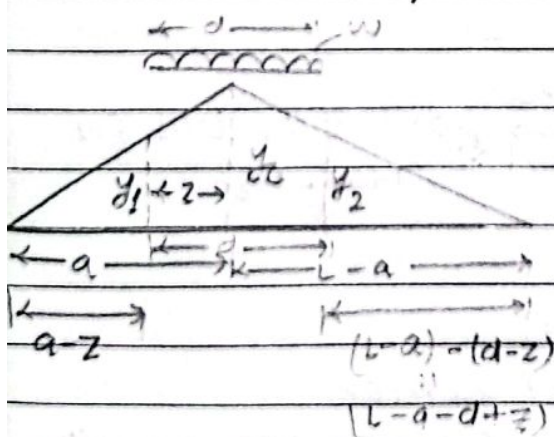


Max^m BM_C,

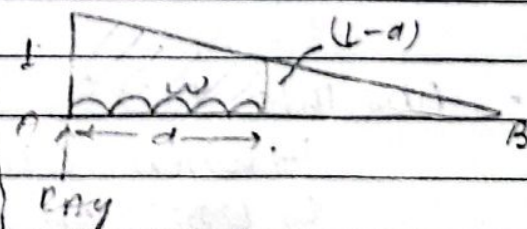
$$y_c = \frac{a(L-a)}{L}$$



Max^m BM_C will occur
 when UDL is acting
 on either side of C



Abs. max^m SF



$$(M_C)_{\max} = w \left[\frac{(y_c + y_1)z}{2} + \frac{(y_c + y_2)(d-z)}{2} \right]$$

$z \rightarrow$ varying
For $(M_c)_{\max}$,

For given, $y_c \Rightarrow y_1, y_2$
are fixed.

$$\frac{dM_c}{dz} = 0$$

$$\text{or } \frac{w}{2} \left[\frac{y_c + y_1}{2} - \frac{y_c + y_2}{2} \right] = 0$$

$$\text{or } y_1 = y_2 \} \rightarrow \text{ordinates } y_1 \text{ \& } y_2 \text{ equal.}$$

$$\text{or } \frac{y_c \times (a - z)}{a} = \frac{y_c}{(L - a)} \times (L - d - a + z) \quad \left\{ \begin{array}{l} \text{From similar} \\ \Delta \end{array} \right.$$

$$\text{or } \frac{a - z}{a} = \frac{L - d - a + z}{L - a}$$

$$\text{or } La - LZ - a^2 + az = La - ad - a^2 + az$$

$$\text{or } La - LZ = La - ad$$

$$\text{or } LZ = ad$$

$$\Rightarrow z = \left[\frac{z}{d} = \frac{a}{L} \right]$$

$$\left[z = \left(\frac{a}{L} \right) d \right] \rightsquigarrow \text{position of UDL for } \text{Max. } M_c$$

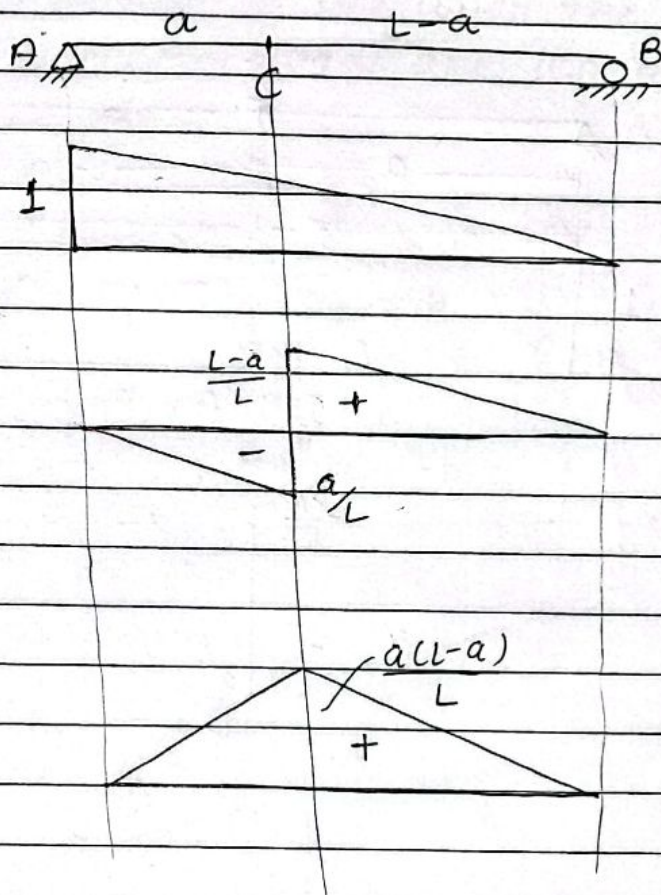
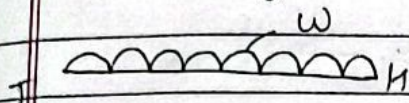
• Absolute Max^m B.M.,

↳ when C is at center

↳ UDL spread both side equally

$$\left\{ \begin{array}{l} \text{i.e. } z = \frac{d}{2} \\ \& a = \frac{L}{2} \end{array} \right.$$

(3) # UDL longer than span



$$(R_A)_{\max} = \frac{wL}{2} [w \times \text{Area under UDL}]$$

$$(R_B)_{\max} = \frac{wL}{2}$$

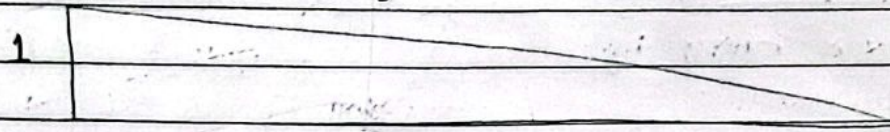
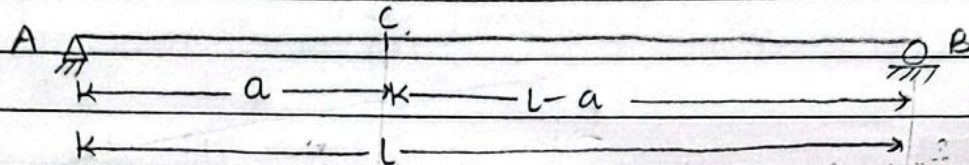
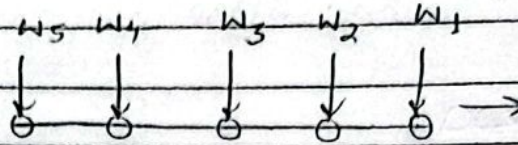
Max -ve SF at C will occur when head of UDL is at C $\left(\frac{1}{2} \times a \times \frac{a}{L} \times w \right) = \left(\frac{wa^2}{2L} \right)$

Max +ve SF at C will occur when tail of UDL is at C. $\frac{1}{2} \times (L-a) \times \left(\frac{L-a}{L} \right) \times w = \left(\frac{w(L-a)^2}{2L} \right)$

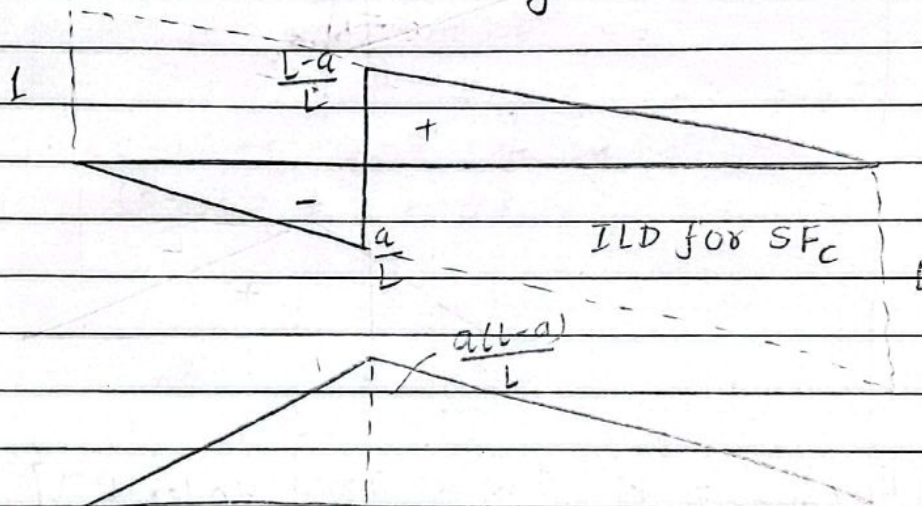
Max BM at C will occur when UDL is whole of the span. $\frac{1}{2} \times L \times \frac{a(L-a)}{L} \times w = \frac{wa(L-a)}{2}$

Abs. Max BM will occur when ~~at~~ C is at middle of the span and UDL is at whole part of span. $\frac{1}{2} \times L \times \frac{L}{2} \times w = \frac{wL^2}{4}$

(4) Several concentrated loads



ILD for R_A .



ILD for M_C

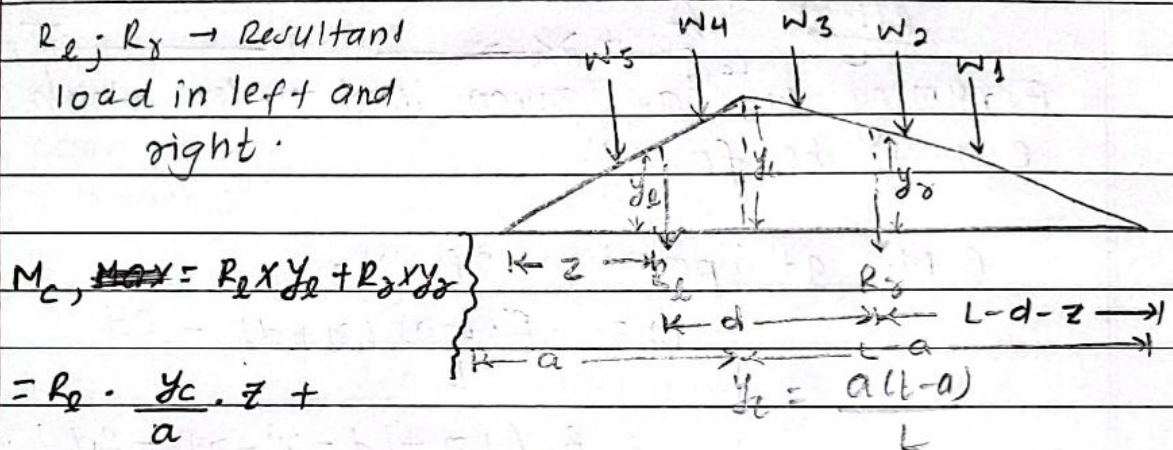
✖ Max R_A when W_5 is at A (unless magnitude of other loads is much larger than W_5 : Total need to be done).

✖ Abs. Max^(+ve) SF when, C pt is at A. (& W_5 at A)
Abs. Max^(-ve) SF , when C pt is at B. (& W_1 at B)

Imp. $\text{Max}^m \text{ BM at } C = ?$
 $\text{Abs. max}^m \text{ BM} = ?$

- $\text{Max}^m \text{ BM at } C$ will occur when the load will be such that some loads are left of C and some loads are right of C .

$R_L, R_R \rightarrow$ Resultant load in left and right.



$$M_C, \text{max} = R_L \cdot y_L + R_R \cdot y_R$$

$$= R_L \cdot \frac{y_L}{a} \cdot z +$$

$$R_R \cdot \frac{y_R}{L-a} \cdot (L-d-z)$$

$$\text{For, } (M_C)_{\text{max}}, \frac{dM_C}{dz} = 0$$

$$\text{or, } \frac{R_L \cdot y_L}{a} - \frac{R_R \cdot y_R}{L-a} = 0$$

$$\text{or, } \left[\frac{R_L}{a} = \frac{R_R}{L-a} \right] \rightarrow \text{for Max. } M_C.$$

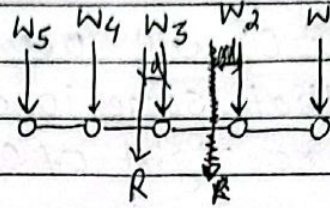
or, Avg. loading on left = Avg. loading on right

$$R_L (L-a) = R_R a$$

$$\text{or, } R_L L - R_L a = R_R a$$

$$\text{or, } \left[R_L = \frac{(R_L + R_R) a}{L} = \frac{W a}{L} \right]$$

* Max^m Bending moment under a given load



Assuming W_3 as given load under which max^m B.M. to be found.

B.M. at position of W_3

$$M = \frac{R(L-x)}{L} (x+d) - Rd$$

$$= \frac{R}{L} (Lx + Ld - x^2 - xd) - Rd$$

For max^m BM,

$$\frac{dM}{dx} = \frac{R}{L} (L - 2x - d) = 0$$

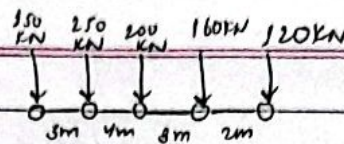
$$L - 2x - d = 0$$

$$x = \frac{L-d}{2} = \frac{L}{2} - \frac{d}{2}$$

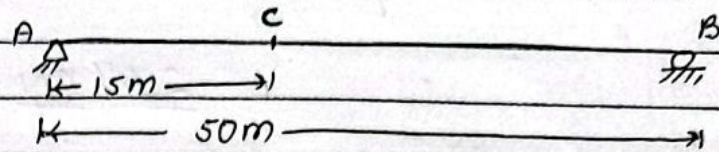
$$\text{Position of } W_3 \text{ (from left)} = x+d = \frac{L}{2} + \frac{d}{2}$$

$$\begin{aligned} \text{Position of } W_3 \text{ from right support} &= L - x - d \\ &= L - \left(\frac{L}{2} - \frac{d}{2} \right) - d \\ &= \frac{L}{2} - \frac{d}{2} \end{aligned}$$

① Determine max^m B.M. at C.



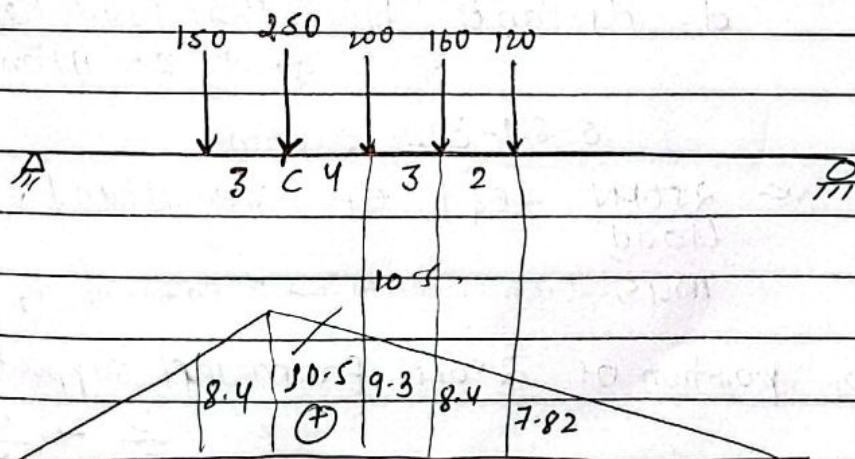
② Determine absolute max^m B.M.



→ Solⁿ:-

For Max^m B.M. at C.

Load passing through C	$\frac{W_L}{a}$	$\frac{W_R}{L-a}$	Remarks
120	$\frac{760}{15}$	$\frac{120}{35}$	$\frac{W_L}{a} > \frac{W_R}{L-a}$
160	$\frac{600}{15}$	$\frac{280}{35}$	$\frac{W_L}{a} > \frac{W_R}{L-a}$
200	$\frac{400}{15}$	$\frac{480}{35}$	$\frac{W_L}{a} > \frac{W_R}{L-a}$
250	$\frac{150}{15}$	$\frac{730}{35}$	$\frac{W_L}{a} < \frac{W_R}{L-a}$ } } change } } (so keep } } 250 } } at C)



ILD for M_C

$$M_{max} = 8025 \text{ kNm}$$

or

$$W_L = \frac{W \cdot a}{L} = \frac{(150 + 250 + 200 + 160 + 120) \times 15}{50}$$

$$= 264 \text{ KN (load on left of C)}$$

So, 250 KN load on C gives max^m load BM at C

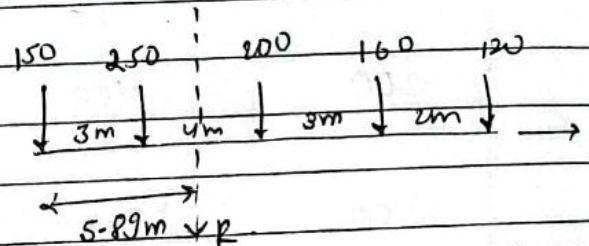
* Absolute Max^m BM:-

① Find position of resultant:-

(of load system) - from 150 KN load

$$\bar{x} = \frac{150 \times 0 + 250 \times 3 + 200 \times 7 + 160 \times 10 + 120 \times 12}{880 \text{ (total load)}}$$

$$= 5.89 \text{ m.}$$



~ Find max^m BM on either side of resultant

② • Find BM_{max} under the load 250 KN.
d = distance b/w that load (here 250 KN) and resultant (r)

$$= 5.89 - 3 = 2.89 \text{ m.}$$

~ 250 KN left of resultant;
(load)

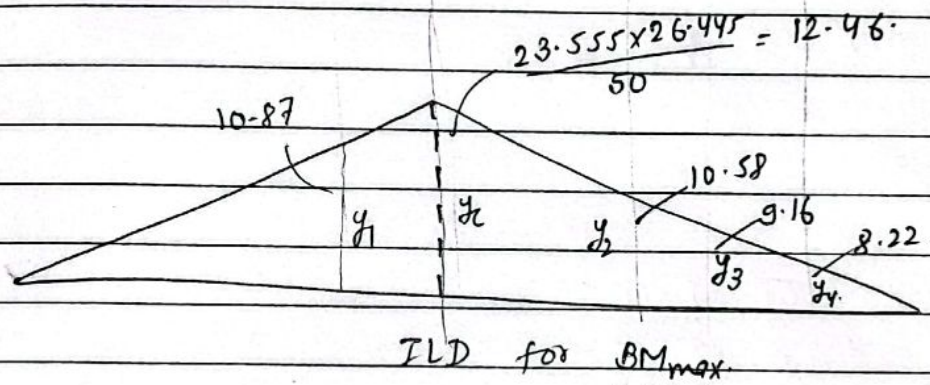
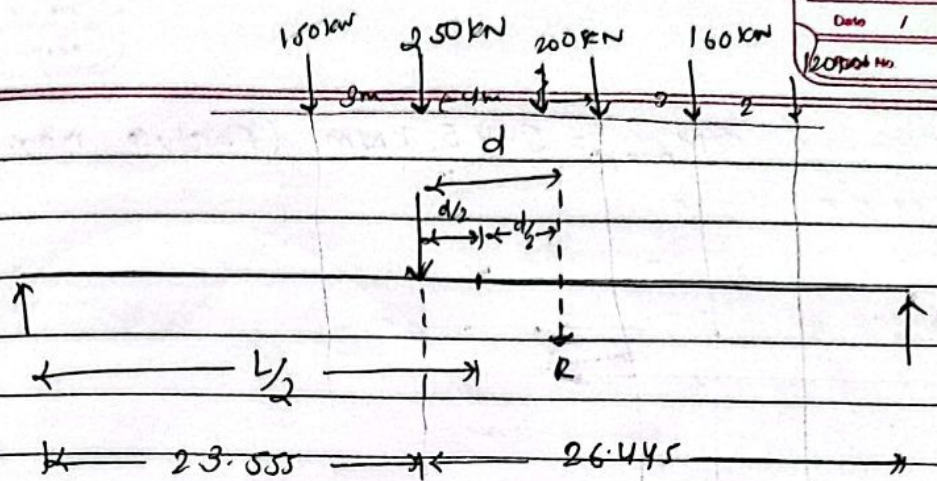
Thus, (3rd load is max^m for 1st, 2nd, & ...)

So, position of 250 KN from left support is

$$= \frac{L}{2} - \frac{d}{2}$$

$$= \frac{50}{2} - \frac{2.89}{2}$$

$$= 23.555 \text{ m. (keep 250 at this position from left)}$$



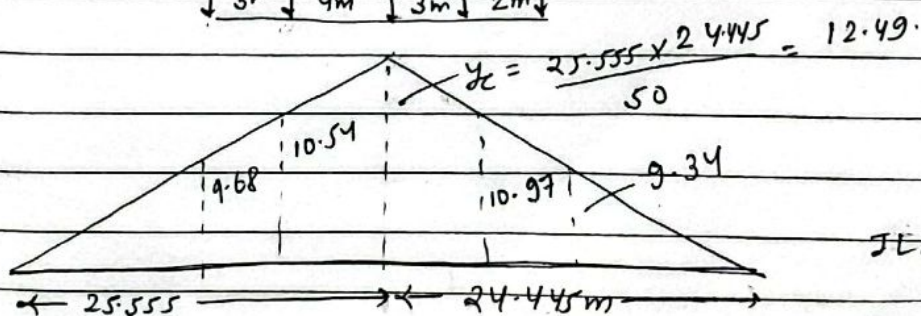
$$BM_{max} = 9313.5 \text{ kNm}$$

③ Find BM_{max} under the load 200 kN.

$$d = \text{distance b/w } 200 \text{ kN load and resultant} \\ = 7 - 5.89 = 1.11 \text{ m.}$$

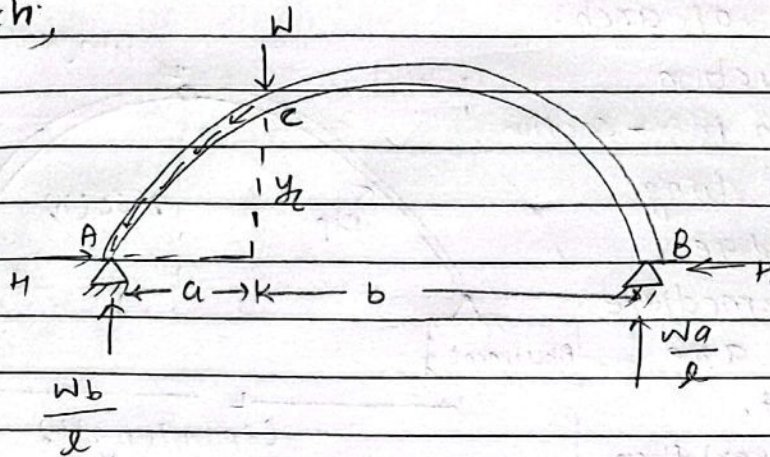
$$\text{Position of } 200 \text{ kN load} = \frac{L}{2} + \frac{d}{2}$$

$$= 25 + \frac{1.11}{2} = 25.555 \text{ m.}$$

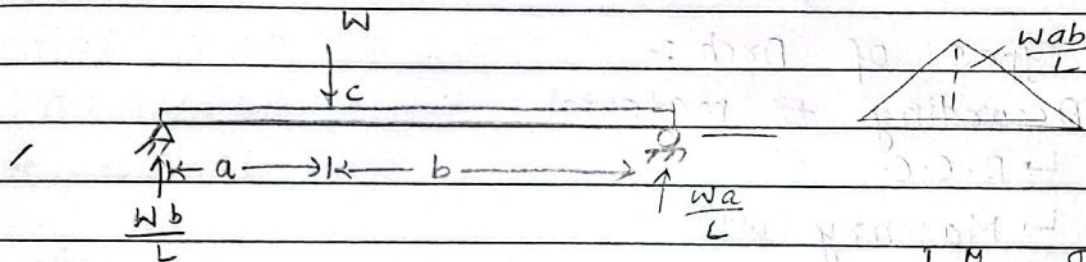


$$BM_{max} = 9445 \text{ kNm (Absolute max. BM).}$$

Arch,



Beam:



For Beam, $M_c = \frac{Wab}{L}$

For Arch $M_c = \frac{Wb}{L} a - Hy$

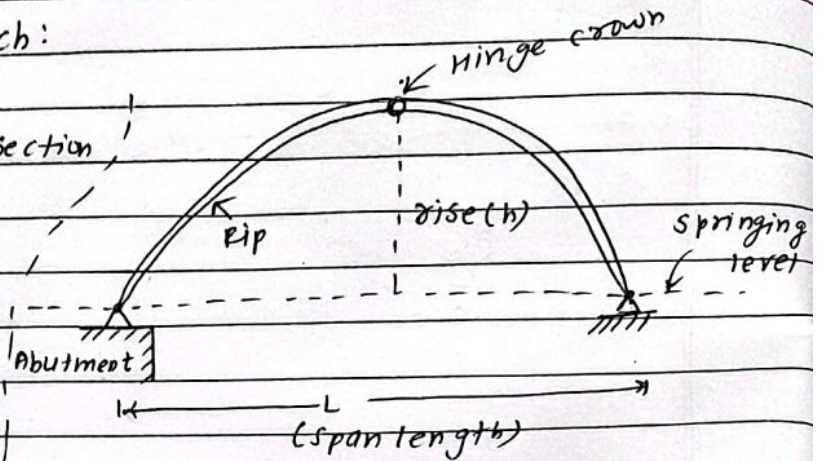
BM reduced

(less cross-section, less cost however construction cost more)

BM on arch is less than that of beam. So, less X-section is enough in arch than in beam.

Advantage of arch:

- i) BM reduction
- ii) Reduction in x-section
- iii) Used in long span Bridge
- iv) If intermediate supports are difficult.
eg. (deep river) then, Arch bridge is used



Types of Arch :-

• According to material

- ↳ R.C.C.
- ↳ Masonry (wall)
- ↳ Steel Arch

• According to shape

- ① Parabolic - easy in calculation
- ② Circular

• According to structural property

- ↳ statically determinate $\Rightarrow$ Three-hinged Arch
- ↳ statically indeterminate $\Rightarrow$ Two-hinged Arch.
 - $\hookrightarrow$ Hingeless / Fixed Arch.

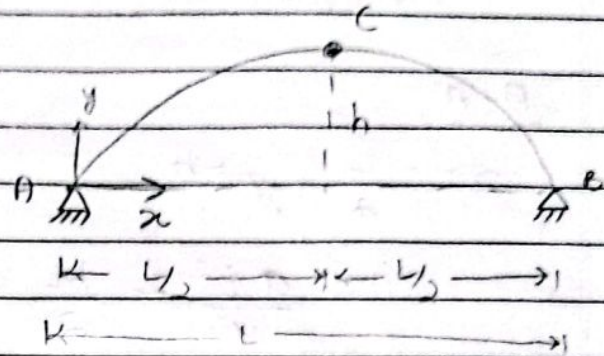
Analysis of Arch:-
shape $\rightarrow$ Parabolic

Equation: $y = ax^2 + bx + c$

• Taking A as origin,
 $x=0, y=0, c=0$

At C,

$x = L/2, y = h$



$$h = a \frac{L^2}{4} + \frac{bL}{2} \quad \text{--- (1)}$$

At B,

$x = L, y = 0$

$0 = aL^2 + bL$

or $bL = -aL^2 \quad \text{--- (2)}$

From (1) & (2)

$$h = \frac{aL^2}{4} - \frac{aL^2}{2} = -\frac{aL^2}{4}$$

$$\left[a = \frac{-4h}{L^2} \right] \left[b = -a \times L = \frac{4h}{L} \right]$$

Thus, Equation of parabolic arch:

$$y = \left(\frac{-4h}{L^2} \right) x^2 + \left(\frac{4h}{L} \right) x$$

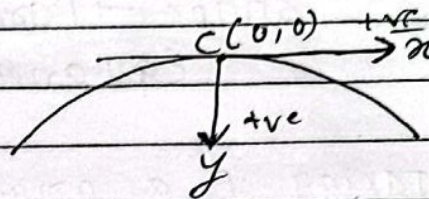
$$\left[y = \frac{4hx(L-x)}{L^2} \right] \text{ or } \cdot$$

$\rightarrow$ to calculate ordinate of each points on arch.
 $\rightarrow$ also to find angle made by tangent at any pt- on arch. $\left(\frac{dy}{dx} = \tan \theta \right)$

$\theta \rightarrow$ angle made with horizontal.

• If C is origin

At C, $x=0, y=0$
So, $c=0$



At A,

$$x = -\frac{L}{2}, y = h$$

$$h = \frac{aL^2}{4} - \frac{bL}{2} \quad \text{--- (i)}$$

At B,

$$x = +\frac{L}{2}, y = h$$

$$h = \frac{aL^2}{4} + \frac{bL}{2} \quad \text{--- (ii)}$$

$$\text{(i) + (ii)}$$

$$2h = \frac{aL^2}{4} \times 2$$

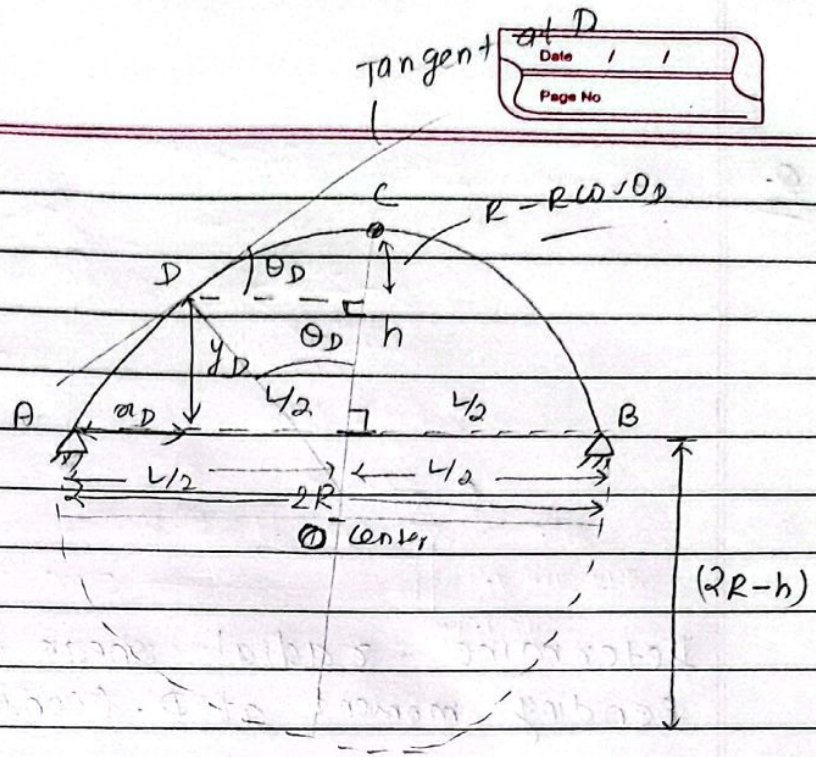
$$\left[a = \frac{4h}{L^2} \right] \quad \& \quad [b=0]$$

Thus,

$$\left[y = \frac{4h}{L^2} x^2 \right] \underline{\underline{u}}$$

Similar to $y = kx^2$

Circular Arch



From property of circle,

$$\frac{L}{2} \times \frac{L}{2} = (2R-h) \times h$$

$$\Rightarrow \left[R = \frac{L^2}{8h} + \frac{h}{2} \right] \text{ w. }$$

$$\left[\sin \theta_D = \frac{\frac{L}{2} - x_D}{R} \right] \text{ w. }$$

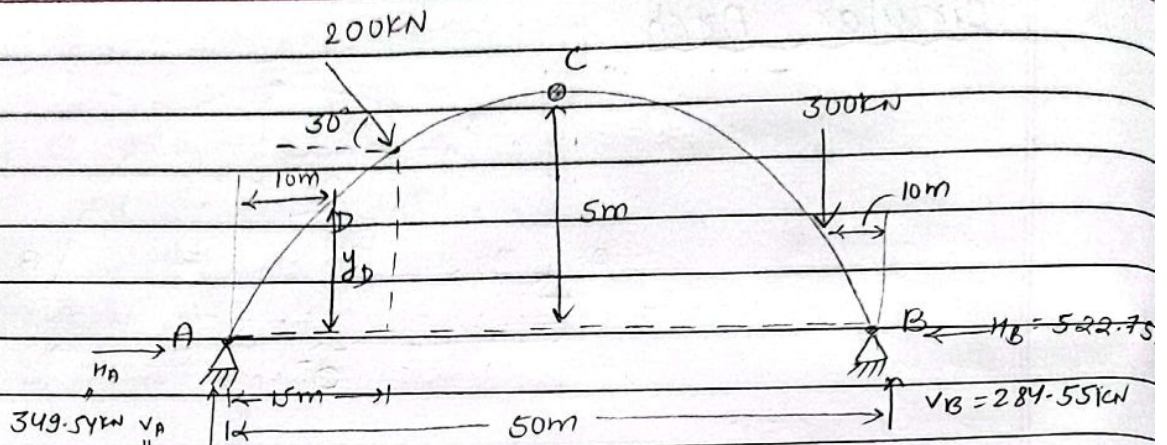
$$\theta_D = \theta_C$$

} Need to find θ for each pt.

$$y_D = h - (R - R \cos \theta_D)$$

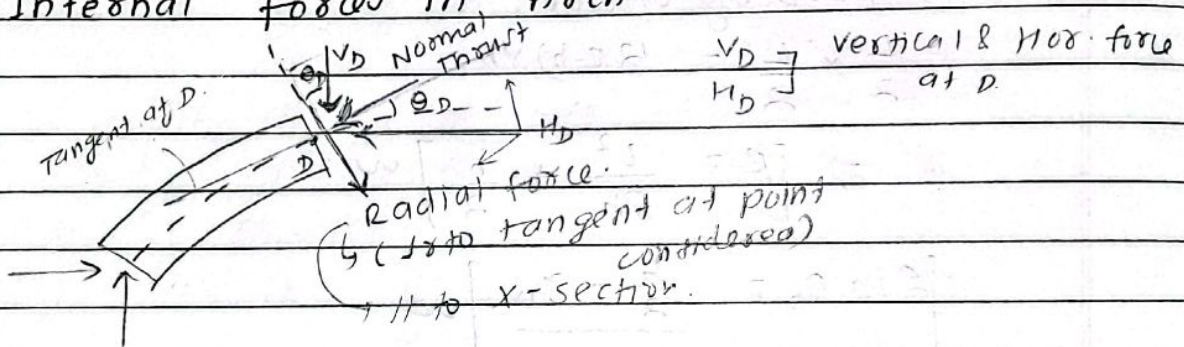
$$\left[y_D = h - R(1 - \cos \theta_D) \right] \text{ w. }$$

Q.



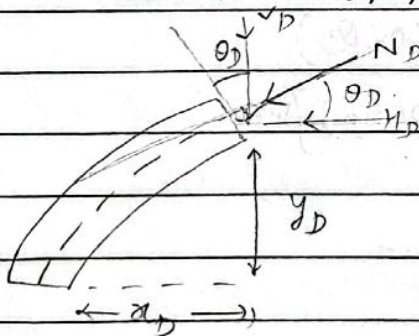
Determine radial shear force, normal thrust and Bending moment at D. (Considering both circular & parabolic arch)

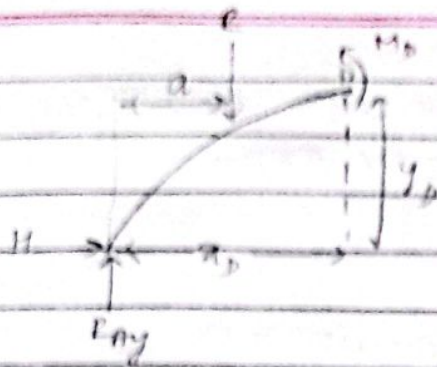
• Internal forces in Arch



$$\text{Radial shear force } (Q_D) = V_D \cos \theta_D - H_D \sin \theta_D$$

$$\text{Normal thrust } (N_D) = V_D \sin \theta_D + H_D \cos \theta_D$$





$$M_D = [R \sin \theta_D \times a_D - R (a_D - a)] - H \times y_D$$

$$[M_D = \underbrace{(M_D)_{\text{beam}}}_{M_D} - H y_D]$$

Parabolic

$$y = \frac{4h\alpha}{L^2} (L - \alpha)$$

$$y_{10} = y_D = \frac{4 \times 5 \times 10}{50^2} (50 - 10) = 3.2 \text{ m}$$

$$y_{15} = \frac{4 \times 5 \times 15}{50^2} (50 - 15) = 4.2 \text{ m}$$

$$\frac{dy}{d\alpha} = \tan \theta_D = \frac{4h}{L^2} (L - 2\alpha)$$

For y_{10} , ($\alpha = 10 \text{ m}$)

$$\theta_D = 13.49^\circ$$

$$\sum M_A = 0 (\curvearrowright +ve)$$

$$\text{or, } 200 \sin 30^\circ \times 15 + 200 \cos 30^\circ \times 4.2 + 300 \times 40 - V_B \times 50 = 0$$

$$\Rightarrow V_B = 284.55 \text{ kN.}$$

$$\sum F_y = 0 \uparrow$$

$$\text{or, } V_A + V_B - 200 \sin 30^\circ - 300 = 0$$

$$\text{or, } V_A = 115.45 \text{ kN}$$

$$M_C = 0 \text{ from right:}$$

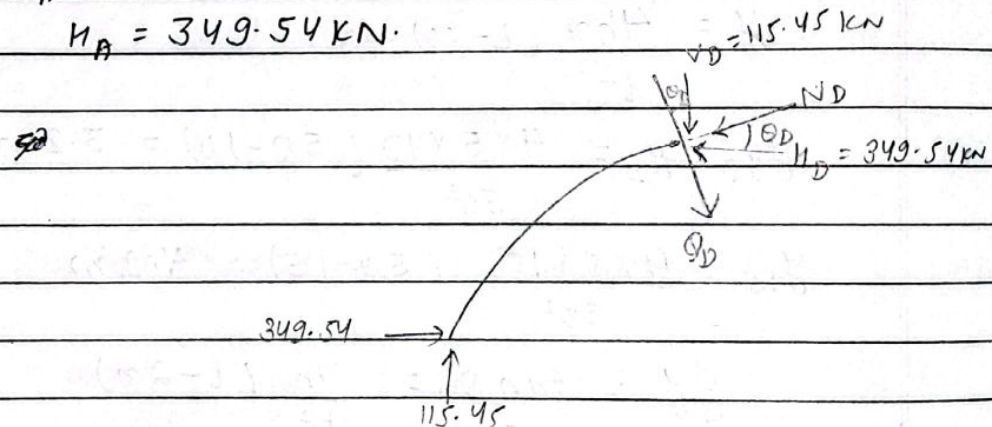
$$\text{or, } H_B \times 5 + 300 \times 15 - 284.55 \times 25 = 0$$

$$[H_B = 522.75 \text{ kN}]$$

$$\sum F_x = 0 (\rightarrow +ve)$$

$$\text{or, } H_A - 522.75 + 200 \cos 30^\circ = 0$$

$$\text{or, } H_A = 349.54 \text{ kN.}$$



$$Q_D = V_D \cos \theta_D - H_D \sin \theta_D = 115.45 \cos(13.89^\circ) - 349.54 \sin(13.89^\circ)$$

$$= 30.726 \text{ kN}$$

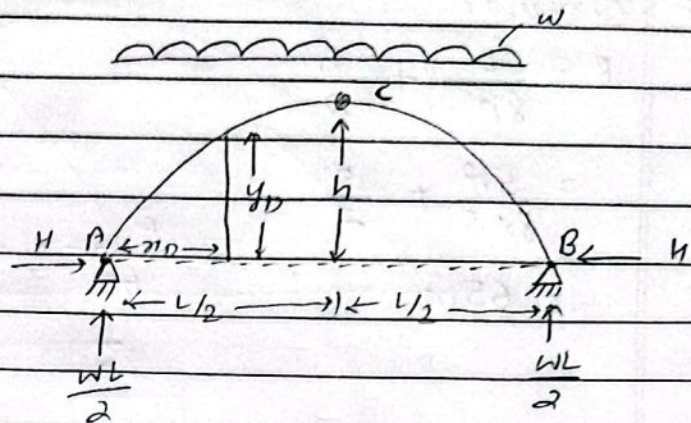
$$N_D = V_D \sin \theta_D + H_D \cos \theta_D = 115.45 \sin(13.89^\circ) + 349.54 \cos(13.89^\circ)$$

$$= 366.828 \text{ kN}$$

$$M_D = 115.45 \times 10 - 349.54 \times 3.2$$

$$= 10.35.972 \text{ kNm.}$$

* Three hinged Parabolic Arch with UDL on its whole span.



$$M_C = \frac{WL}{2} \times \frac{L}{2} - w \times \frac{L}{2} \times \frac{L}{4} - H \times h = 0$$

(right part)

$$\left[H = \frac{WL^2}{8h} \right] \checkmark$$

(BM)

$$M_D = \frac{WL}{2} \times x_D - w x_D \times \frac{x_D}{2} - H \times y_D$$

$$= \frac{WL}{2} \cdot x_D - \frac{w x_D^2}{2} - \frac{WL^2}{8h} \times \frac{4h x_D (L - x_D)}{L}$$

$$= 0$$

Radial shear

$$Q_D = V_D \cos \theta_D - H \sin \theta_D$$

$$= \cos \theta_D (V_D - H \tan \theta_D)$$

$$= \cos \theta_D \left\{ \left(\frac{WL}{2} - w \cdot x_D \right) - \frac{WL^2}{8h} \times \frac{4h}{L} (L - 2x_D) \right\}$$

$$= 0$$

dy/dx

2

2

2

2

2

2

2

2

2

2

2

2

2

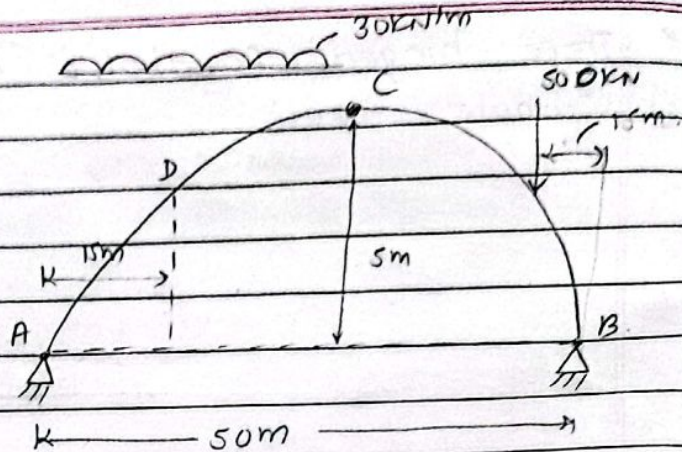
2

x Circular

$$R = \frac{L^2}{8h} + \frac{h}{2}$$

$$= \frac{50^2}{8 \times 5} + \frac{5}{2}$$

$$= 65\text{m}$$



$$\sin \theta_D = \frac{L/2 - x_D}{R}$$

$$= \frac{25 - 15}{65}$$

$$\theta_D = 8.85^\circ$$

$$y_D = h - R(1 - \cos \theta_D) = 4.16\text{m} \quad 4.226\text{m}$$

$$\sum M_A = 0$$

$$30 \times 25 \times 12.5 + 500 \times 35 = V_B \times 50 = 0$$

$$[V_B = 537.5\text{kN}]$$

$$V_A + V_B - 30 \times 25 - 500 = 0$$

$$\text{or } [V_A = 712.5\text{kN}]$$

$$M_C, \text{ right} = 0$$

$$- V_B \times 25 + H \times 5 + 500 \times 10 = 0$$

$$[H = 1687.5\text{kN}]$$

Vertical shear at D

$$V_D = V_A - 30 \times 15 = 262.5 \text{ kN}$$

$$H_D = H = 1687.5 \text{ kN}$$

$$\begin{aligned} Q_D &= V_D \cos \theta_D - H_D \sin \theta_D \\ &= 262.5 \cos(8.85^\circ) - 1687.5 \sin(8.85^\circ) \\ &= -0.244 \text{ kN} \end{aligned}$$

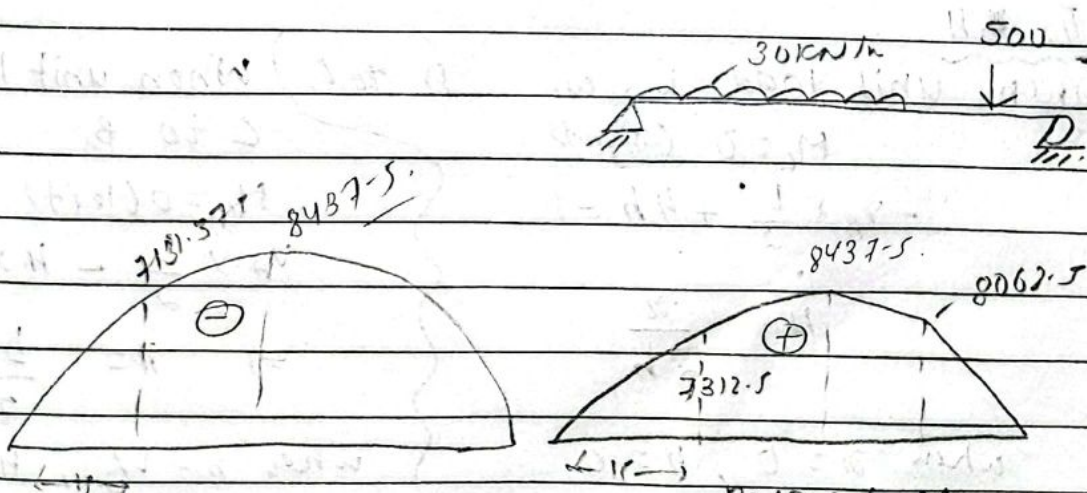
$$\begin{aligned} N_D &= V_D \sin \theta_D + H_D \cos \theta_D \\ &= 262.5 \sin(8.85^\circ) + 1687.5 \cos(8.85^\circ) \\ &= 1707.795 \text{ kN} \end{aligned}$$

$$\begin{aligned} M_D &= M_{D, \text{beam}} - H \times y_D \\ &= \underbrace{V_A \times 15}_{712.5} - \underbrace{30 \times 15 \times \frac{15}{2}}_{a} - 1687.5 \times 4.226 \\ &= 181.125 \text{ kNm} \end{aligned}$$

* TO draw B.M.D.

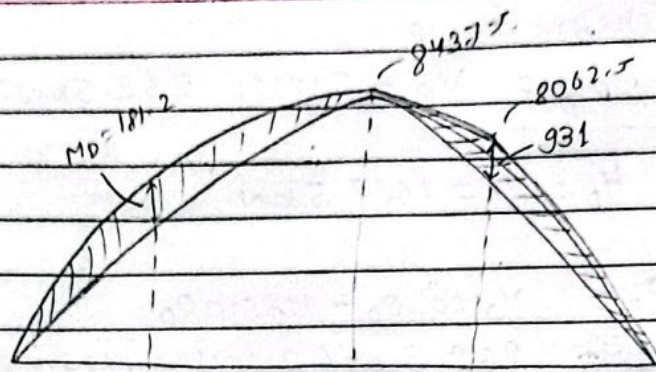
$$y = \frac{4h^2}{L^2} (L-x)$$

$$M_D = M_{D, \text{beam}} - H \times y_D$$



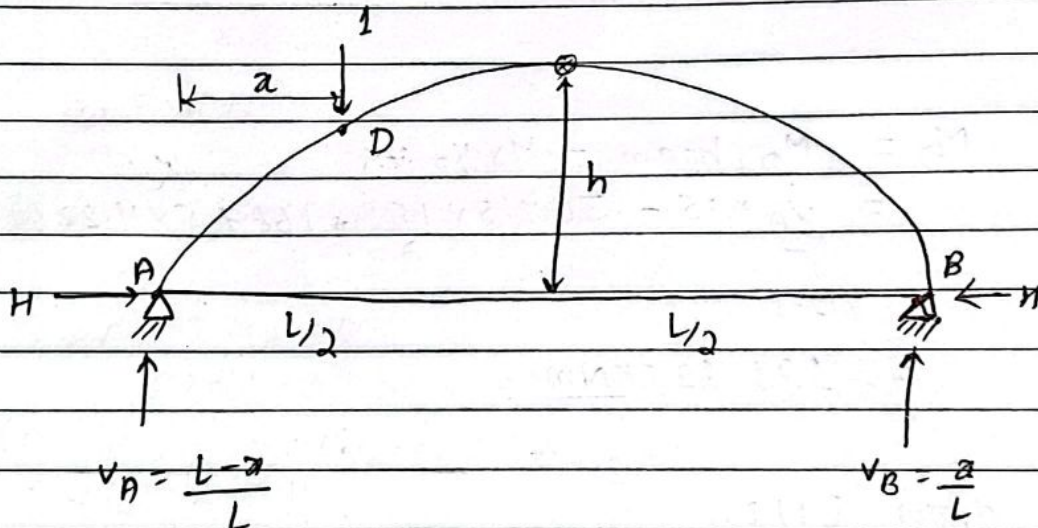
BMD for $H \times y_D$

BMD of M_{beam}



BMD

ILD



For H

when unit load is on A to C,

$M_c = 0$ (right)

$$-V_B \times \frac{L}{2} + H \cdot h = 0$$

$$H = \frac{a}{2h}$$

when, $a = 0$, $H = 0$

when $a = \frac{L}{2}$, $H = \frac{L}{4h}$

When unit load is on C to B,

$M_c = 0$ (left)

$$V_A \times \frac{L}{2} - H \times h = 0$$

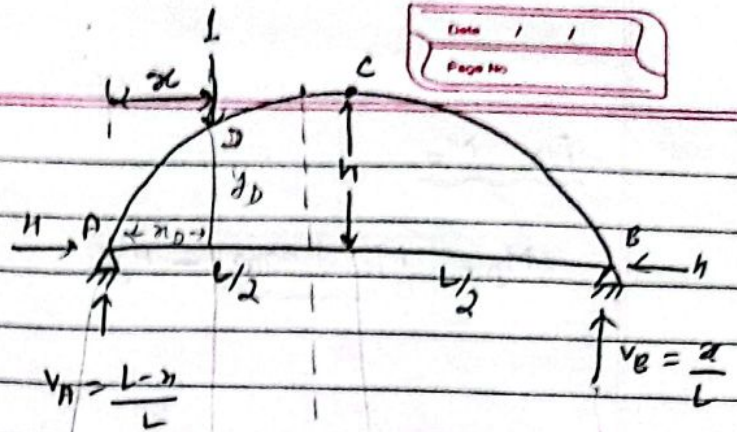
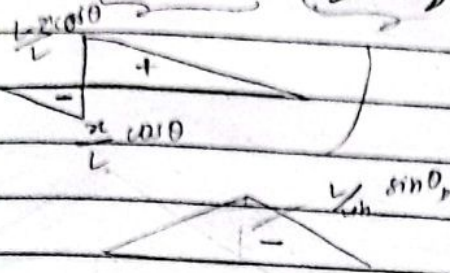
$$\Rightarrow H = \frac{L-a}{2h}$$

when $a = \frac{L}{2}$, $H = \frac{L}{4h}$

when $a = L$, $H = 0$

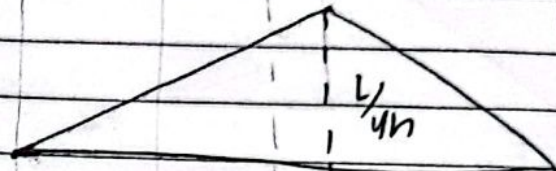
For Radial shear

$$Q_D = V_D \cos \theta_D - H \sin \theta_D$$

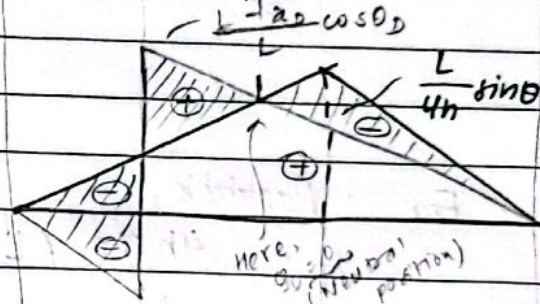


For Normal Thrust

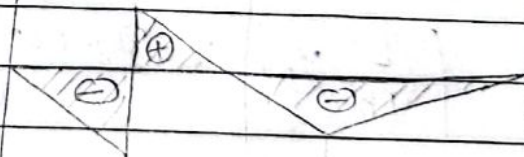
$$N_D = V_D \sin \theta_D + H \cos \theta_D$$



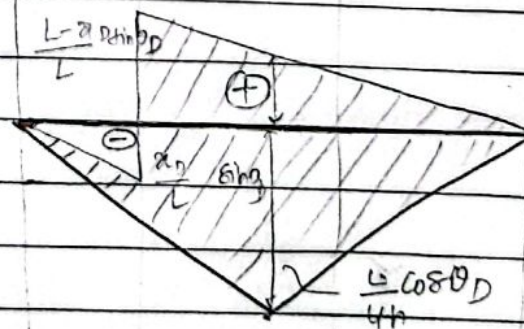
ILD for H



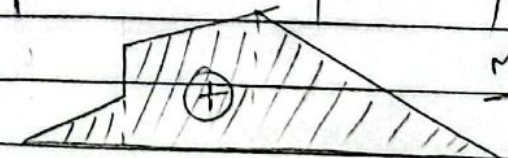
ILD for Q_D



Merging



(No any position of $N_D = 0$ for arch)

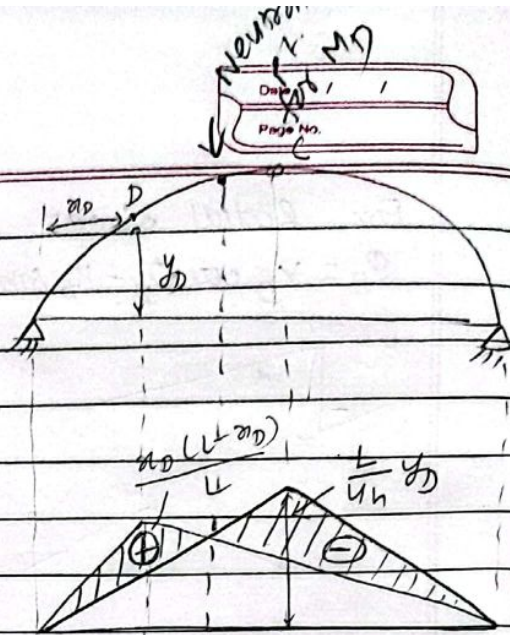
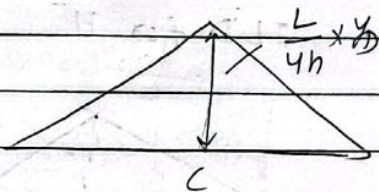
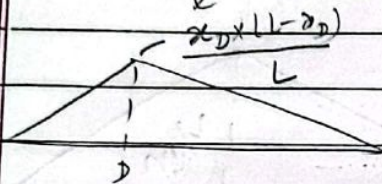


Merging

ILD for N_D

For M_D

$$M_D = \underbrace{M_{D, \text{Beam.}}}_{x_D \times (L - x_D)} - \underbrace{H y_D}_{\frac{L}{4h} x_D}$$



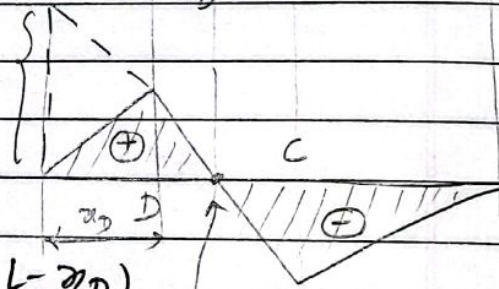
ILD for M_D

Neutral pt.
($M_D = 0$)

For Parabolic,

$$y_D = \frac{4h x_D (L - x_D)}{L^2}$$

$$\frac{L}{4h} y_D = \frac{L}{4h} \left(\frac{4h x_D}{L^2} (L - x_D) \right)$$



Neutral point for x_D

$$\left[\frac{L}{4h} y_D = \frac{x_D (L - x_D)}{L} \right] \rightarrow \text{for parabolic same ht.}$$

* Arch with supports at different level

If C is origin,
 $y = Kx^2$

For B,
 $x = L_2$
 $y = h_2$

$$h_2 = K L_2^2 \quad \text{--- (I)}$$

For A,
 $x = -L_1$
 $y = h_1$

$$h_1 = K L_1^2 \quad \text{--- (II)}$$

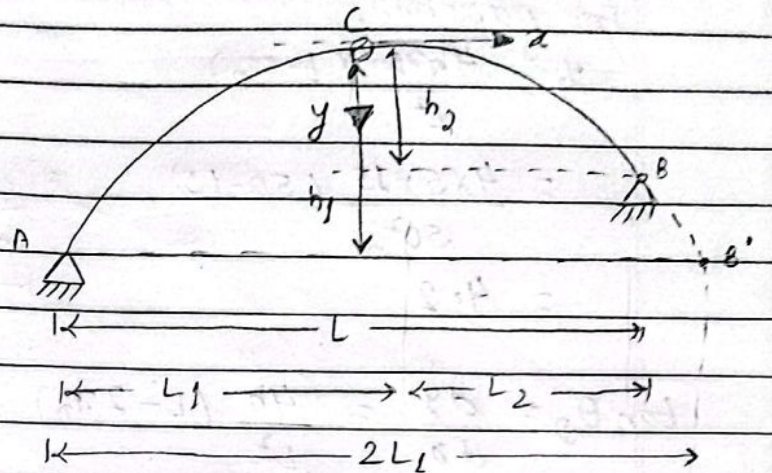
$$\frac{h_1}{h_2} = \frac{L_1^2}{L_2^2}$$

$$\frac{L_1}{L_2} = \sqrt{\frac{h_1}{h_2}}$$

$$\frac{L_1 + L_2}{L_2} = \frac{\sqrt{h_1} + \sqrt{h_2}}{\sqrt{h_2}} \quad \text{[+1 on both side]}$$

$$\left[L_2 = \frac{L \sqrt{h_2}}{\sqrt{h_1} + \sqrt{h_2}} \right] \checkmark$$

$$[L_1 = L - L_2] \checkmark$$



Find ϕ_D , N_D & M_D using ILD

→ Solⁿ:-

For parabola,

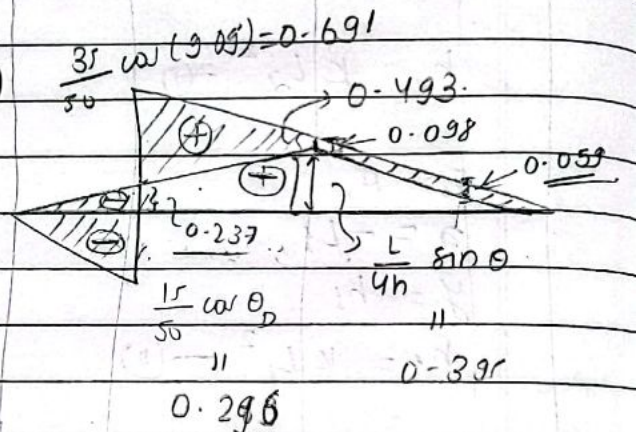
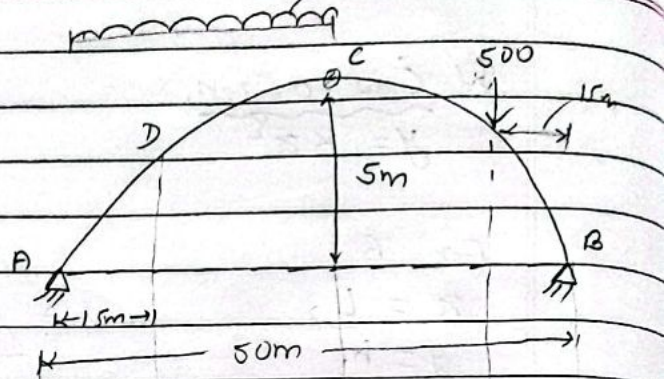
$$y = \frac{4hx}{L^2} (L - x)$$

$$= \frac{4 \times 5 \times 15}{50^2} (50 - 15)$$

$$= 4.2$$

$$\tan \theta_D = \frac{dy}{dx} = \frac{4h}{L^2} (L - 2x)$$

$$[\theta_D = 9.09^\circ]$$



$$\phi_D = 30 \times \left[\frac{-1}{2} \times 15 \times 0.533 \right.$$

$$\left. + \frac{1}{2} \times (0.454 + 0.098) \times 10 \right]$$

$$+ 500 \times 0.059$$

$$= -7.625 \text{ kN}$$

ILD for ϕ_D

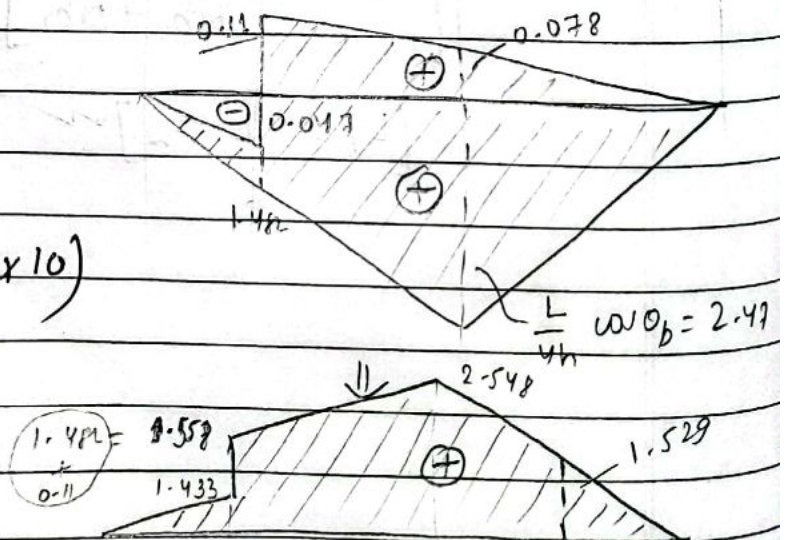
$$N_D = 30 \times \left(\frac{1}{2} \times 15 \times 1.433 \right.$$

$$+ 10.137$$

$$\left(\frac{1.558 + 1.433}{2} \right) \times 10$$

$$+ 500 \times 1.529$$

$$= 1702.825 \text{ kN}$$



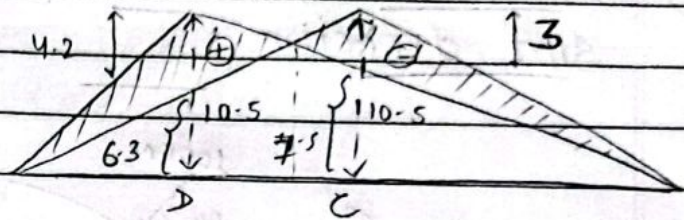
ILD for N_D

Q

$$\frac{4.2}{15} = 4.2$$

$$\alpha = 15$$

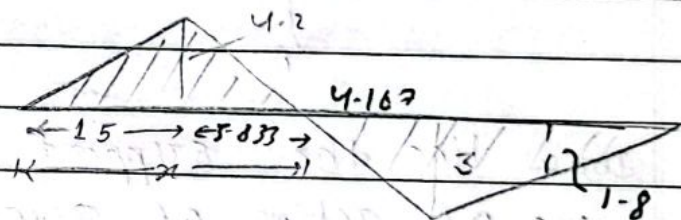
$$\pi = 20.833 \text{ m.}$$



$$M_D = 30 \times \left[\frac{1}{2} \times 20.833 \times 4.2 \right. \\ \left. - \frac{1}{2} \times 4.167 \times 3 \right]$$

$$- 500 \times 1.8$$

$$= 224.964 \text{ Nm}$$

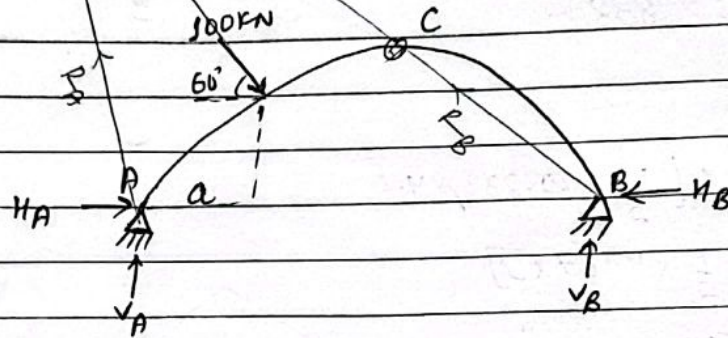


ILD for M_D

* Graphical Method

dotting for each force

- ① Draw Arch with suitable scale.
- ② Locate the given load with proper position and direction.



- (3) Draw line thr. support and hinge C to meet line of action of given force (Resultant at point thr. C)

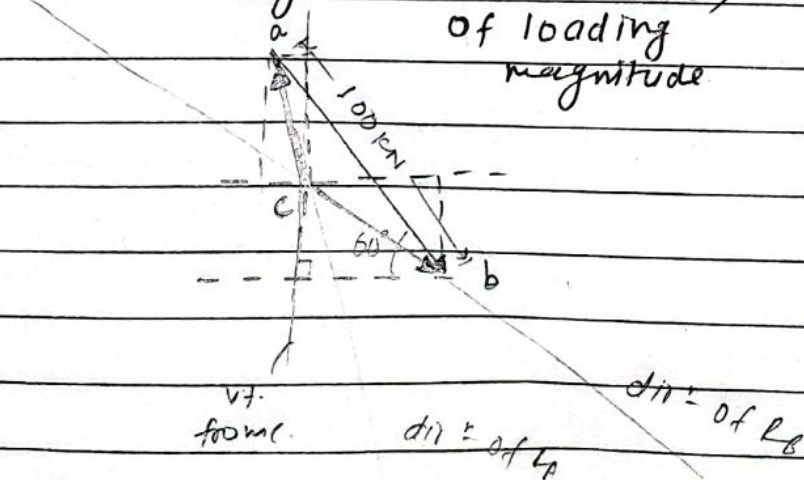
↳ This line

↳ Represent dirⁿ of R_B (Resultant at pt. B)

↳ This line represent line of action of R_B .

- (4) Draw line from support A to meet the intersection pt. of force (applied) & R_B
↳ This line represents line of action of R_A .

- (5) Draw a vector representing given force (near to given pt.)
(length = acc. to scale)



(6) Draw lines parallel to R_B & R_A from the end points of force vector.

[It forms force triangle, each side gives magnitude & dirⁿ of R_B & R_A]

$$\left. \begin{aligned} bc &= \vec{R}_B \\ ca &= \vec{R}_A \\ ba &= \text{applied.} \end{aligned} \right\} \text{Magnitudes}$$

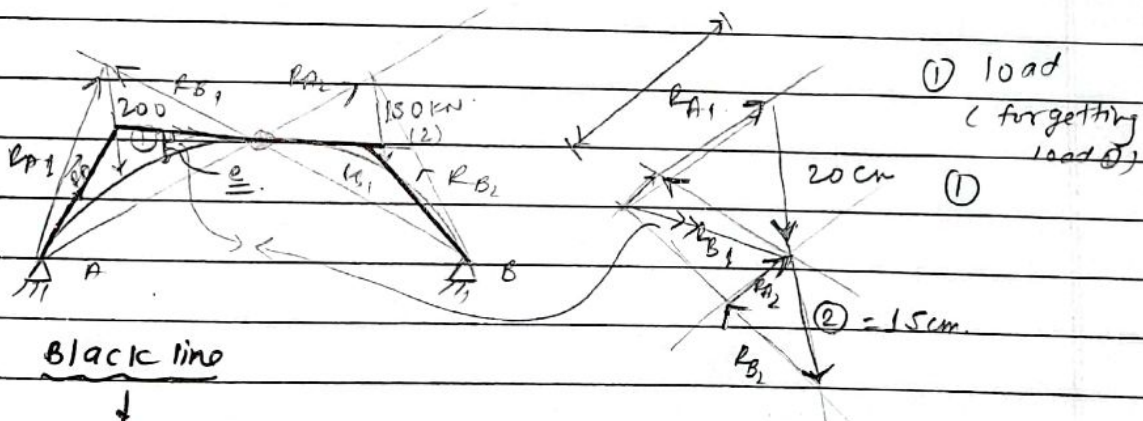
(7) Draw vertical & horizontal line at pt. C.

↳ Projection of R_A on vertical line = V_A

Projection of R_A on hor. line = H_A .

Also, projection of R_B on horizontal line = H_B
" " vertical " = V_B .

~ For vertical force

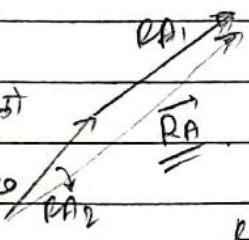


Black line

Line of thrust ~ force लाई vectorially जोड़ने

मां BM
बूटन

→ e → force जो
line में
28 distance
to arch.



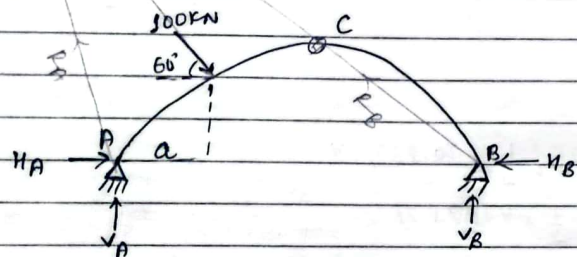
R_{A2} लाई उल्टा दि रखा

R_A को tail में transfer
जाने

* Graphical Method

1 cm = 100 N

- ① Draw Arch with suitable scale.
- ② Locate the given load with proper position and direction.



- ③ Draw line thro. support and hinge C to meet line of action of given force (Resultant at point thro. C)

↳ This line

Represent distⁿ of R_B (Resultant at pt. B)

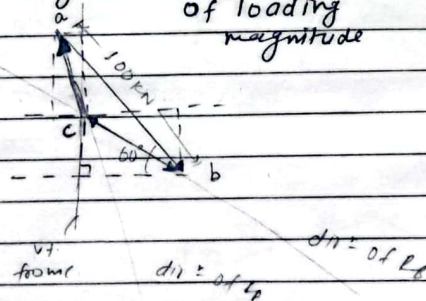
↳ This line represent line of action of R_B

- ④ Draw line from support A to meet the intersection pt. of force (applied) & R_B

↳ This line represents line of action of R_A

- ⑤ Draw a vector representing given force (near to given pt.) (length $\Rightarrow$ acc. to scale)

of loading magnitude



- ⑥ Draw lines parallel to R_B & R_A from the end points of force vector.

(It forms force triangle, each sides gives magnitude (distⁿ of R_B & R_A)

$$bc = R_B$$

$$ca = R_A$$

$$ba = \text{applied.}$$

Magnitude.

- ⑦ Draw vertical & horizontal line at pt. C

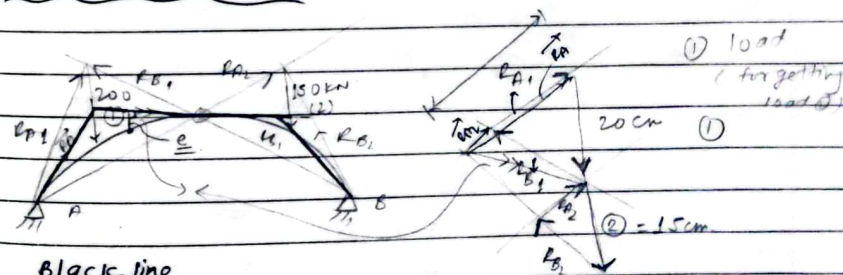
↳ Projection of R_A on vertical line = V_A

Projection of R_A on hor. line = H_A

Also, projection of R_B on horizontal line = H_B

" vertical " = V_B

For vertical force



Black line

Line of thrust & force etc. vertically given

at BM

50 kN

→ e → force at line etc
→ distance to arch

R_A

R_B

R_A

R_B

R_A etc. distance etc. etc.
at pt. C to find total R for

-ve Max. M_D at max = -ve ordinate.

~~See~~

$$M_D^{-ve}, \max = W \left[\frac{x_D(L-x_D)}{L} - \frac{x_D(L-x_D)}{L(L-x_D)} \times \frac{L}{2} \right]$$

$$\frac{dM_D^{-ve}}{dx_D} = 0$$

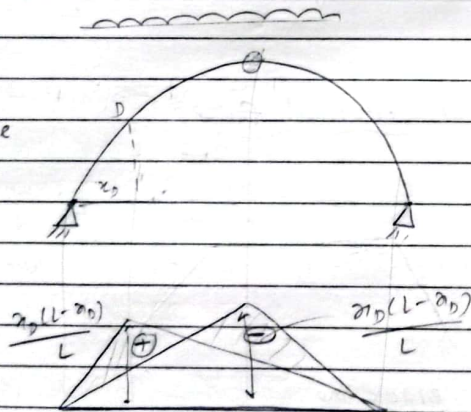
$$\left[x_D = \frac{L}{4} \right] W.$$

$$\left[M_{avr}^{-ve} = \frac{WL}{16} \right] W.$$

* UDL

~ If UDL at whole span, then

$$M_D = 0$$



→ When UDL moves, position of D for abs. Max BM

$$\frac{dM_D}{dx_D} = 0$$

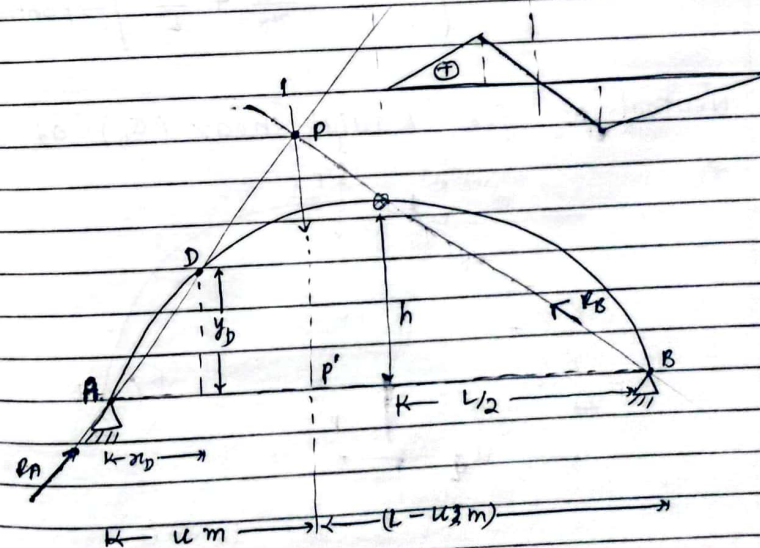
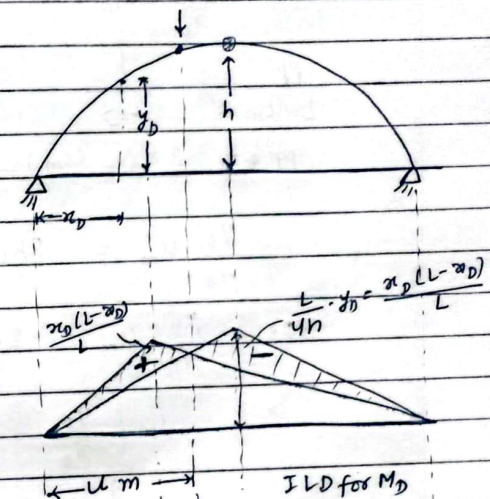
$$\left[x_D = 0.234L \right]$$

$$M_{avr} = \pm 0.01883WL^2$$

* Neutral point

For moment at any point D

Neutral pt. for M_D .
Parabolic.



$$\frac{PP'}{u_m} = \frac{y_D}{x_D}$$

$$PP' = \frac{y_D}{x_D} u_m \quad \text{--- (1)}$$

$$\frac{PP'}{L - u_m} = \frac{h}{L/2}$$

$$PP' = \frac{2h(L - u_m)}{L} \quad \text{--- (2)}$$

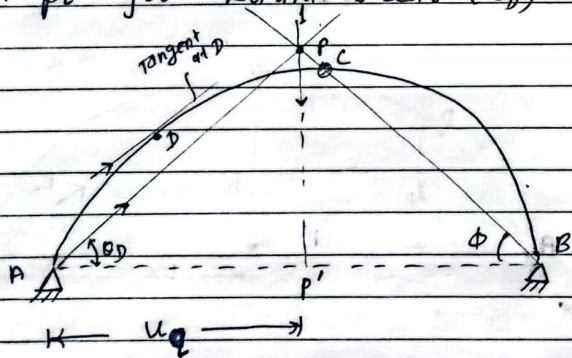
$$\frac{y_D}{x_D} u_m = \frac{2h(L - u_m)}{L}$$

$$\text{or, } \frac{y_D}{x_D} u_m = 2h - \frac{2hu_m}{L}$$

$$\text{or, } u_m \left(\frac{y_D}{x_D} + \frac{2h}{L} \right) = 2h$$

$$\left[u_m = \frac{2h}{\frac{y_D}{x_D} + \frac{2h}{L}} \right] \rightarrow \text{position of neutral pt.}$$

□ Neutral pt. for Radial shear (ϕ_D) at D



$D \rightarrow \phi_D \rightarrow 0$

$\hookrightarrow$ Resultant // to tangent at D

$$\tan \theta_D = \frac{PP'}{u_D}$$

[u_D = pos. of Neutral pt. for ϕ]

$$PP' = u_D \tan \theta_D \quad \text{--- (1)}$$

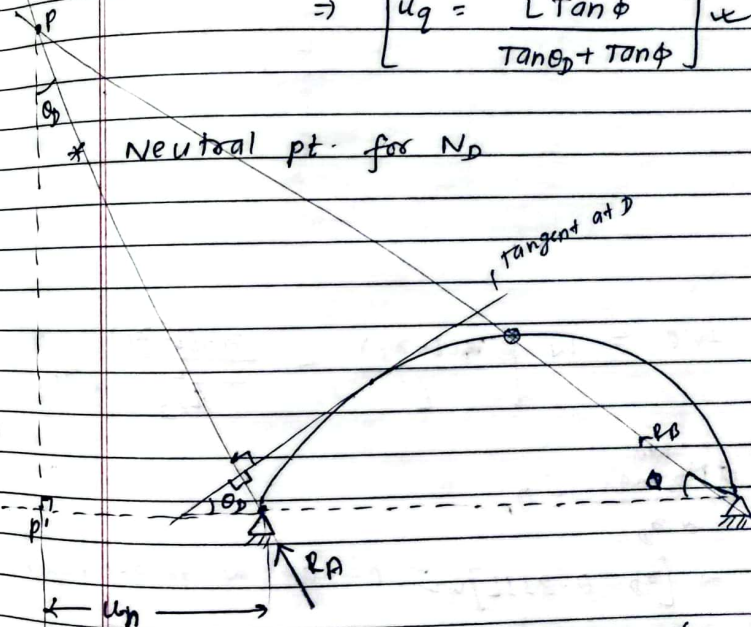
$$\tan \phi = \frac{PP'}{L - u_D}$$

$$PP' = (L - u_D) \tan \phi \quad \text{--- (2)}$$

Equating

$$u_D \tan \theta_D = L \tan \phi - u_D \tan \phi$$

$$\Rightarrow \left[u_D = \frac{L \tan \phi}{\tan \theta_D + \tan \phi} \right] \text{ u.}$$



$$\tan \theta_D = \frac{u_n}{PP'} \Rightarrow PP' = \frac{u_n}{\tan \theta_D} \quad \text{--- (1)}$$

$$\tan \phi = \frac{PP'}{L + u_n} \Rightarrow PP' = (L + u_n) \tan \phi \quad \text{--- (ii)}$$

equating $u_n = (L + u_n) \tan \phi$

$$\tan \theta_D$$

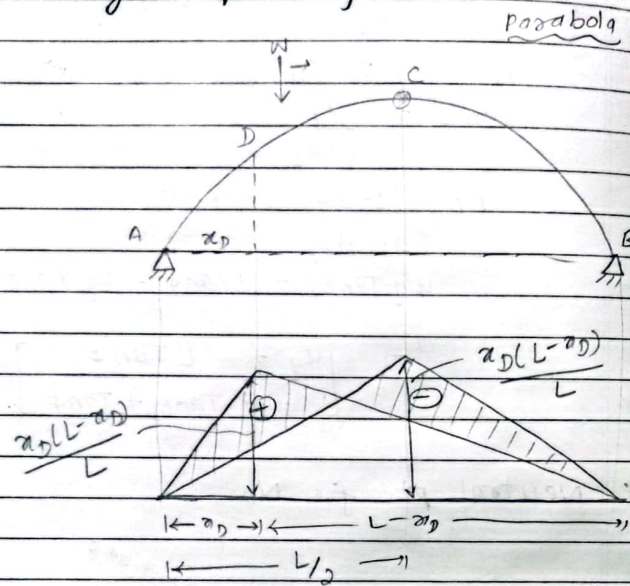
$$\Rightarrow u_n \left[\frac{1}{\tan \theta_D} - \tan \phi \right] = L \tan \phi$$

$$\left[u_n = \frac{L \tan \phi}{1/\tan \theta_D - \tan \phi} \right] \text{ u.}$$

Suspension Cable System

Date / /
Page No.

* Max. B.M. at a given point of arch:



$$M_{D, \max} +ve = W \left[\frac{x_D(L-x_D)}{L} - \frac{x_D(L-x_D)}{L/2} \times \frac{x_D}{2} \right]$$

$$\frac{dM_{D, \max}^{+ve}}{dx_D} = 0$$

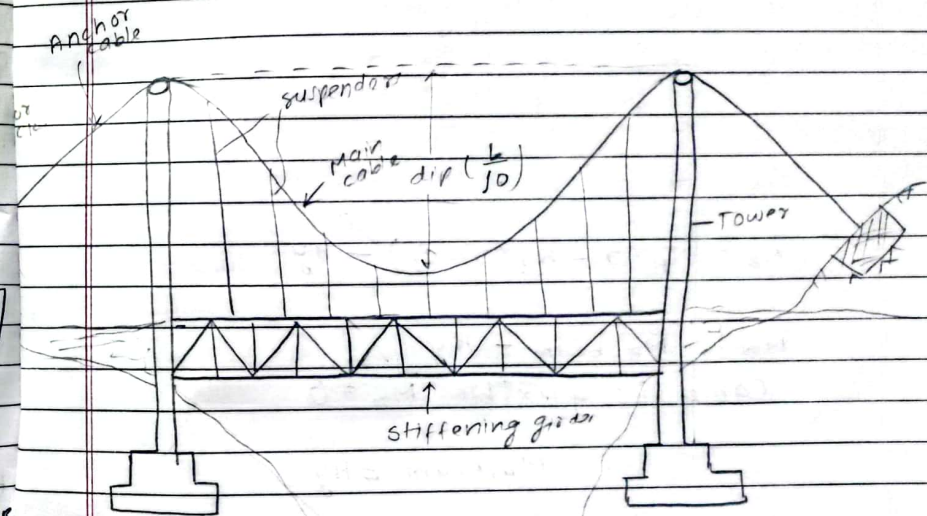
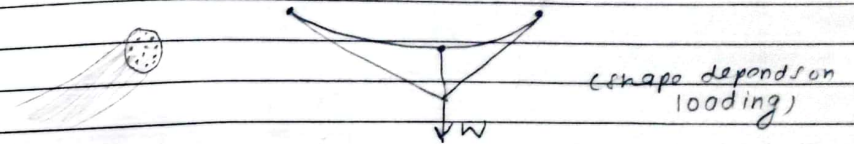
$$\Rightarrow [x_D = 0.211L] \quad \text{--- pos. of } W \text{ for ab. max.}$$

$$[M^{+ve}, \text{abs} = 0.096WL] \quad \text{---}$$

$\Rightarrow$ +ve max. M_D at D इगल है max. +ve ordinate
के position में है।

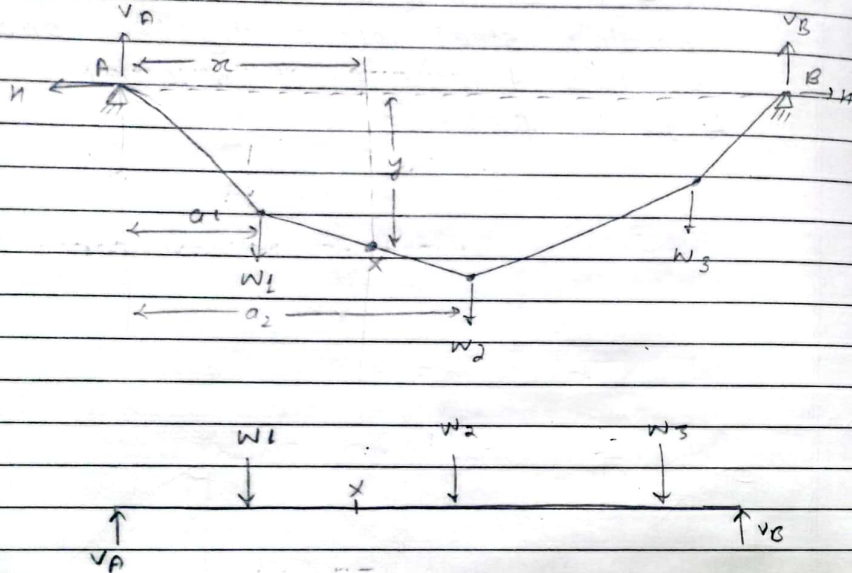
Cable $\Rightarrow$ A structural element made by binding steel wires together and used to take tension force in cable.

$\hookrightarrow$ It is flexible ($BM = 0$)



Fig

* Cable theorem.



$$M_a = V_A \cdot a - w_1(a - a_1) - Hy$$

$$M_a = M_{a, \text{beam}} - Hy$$

Cable is flexible, $M_a = 0$

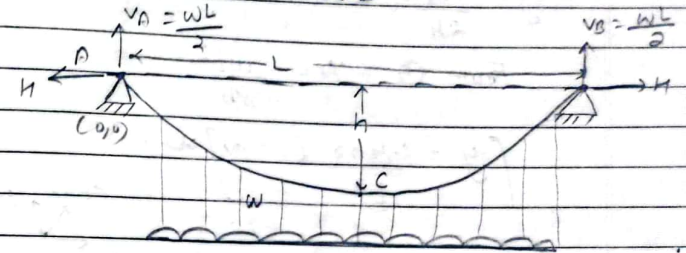
$$\Rightarrow M_{a, \text{beam}} = Hy$$

$$\Rightarrow \left[y = \frac{M_{a, \text{beam}}}{H} \right]$$

∴ This expression determines the shape of cable

Let fix one get another
 $\downarrow$ $\downarrow$
 y_{fix} H_{fix}

* Shape of cable with UDL



$$M_c = 0$$

$$\text{or, } \frac{wL}{2} \times \frac{L}{2} - w \cdot \frac{L}{2} \cdot \frac{L}{2} - H \cdot h = 0$$

$$\text{or, } \left[h = \frac{wL^2}{8H} \right] w$$

∴ For eqⁿ of cable,

$$T \cos \theta = H$$

$$T \sin \theta = \frac{wL}{2} - wa$$

$$\tan \theta = \frac{\frac{wL}{2} - wa}{H}$$

$$\frac{dy}{da} = \frac{\frac{wL}{2} - wa}{H}$$

$$\text{or } y = \frac{wL}{2H} \cdot a - \frac{wa^2}{2H} + C_1 \quad \text{Integrating}$$

Boundary condition,

At A, $a = 0, y = 0$ (origin)

$$\Rightarrow C_1 = 0$$

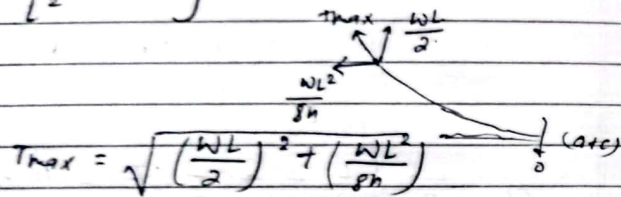
Thus,

$$\left[y = \frac{wL}{2H} a - \frac{wa^2}{2H} \right]$$

$$y = \frac{Wx(L-x)}{8h}$$

From ① $H = \frac{WL^2}{8h}$

$$\left[y = \frac{4h}{L^2} (L-x)^2 \right] x$$



$$T_{max} = \sqrt{\left(\frac{WL}{2}\right)^2 + \left(\frac{WL^2}{8h}\right)^2}$$

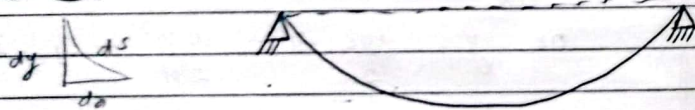
$T_{max} \rightarrow$ at ht.

$T_{min} \rightarrow$ at lowest pt (at C)

$$[T_{min} = H]$$

* length of cable

Support same level



$$ds = \sqrt{dx^2 + dy^2}$$

$$s = \int_0^L \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

$$\frac{dy}{dx} = \left[\tan \theta = \frac{4h}{L^2} (L-2x) \right]$$

$$s = \int_0^L \sqrt{1 + \left[\frac{4h}{L^2} (L-2x) \right]^2} dx$$

$$= \int_0^L \sqrt{1 + \frac{16h^2}{L^4} (L-2x)^2} dx$$

$$= \int_0^L \left[1 + \frac{16h^2}{L^4} (L-2x)^2 \right]^{1/2} dx$$

expand with binomial theorem.

$$= \int_0^L \left[1 + \frac{1}{2} \left(\frac{16h^2}{L^4} (L-2x)^2 \right) + \dots \right] dx$$

neglecting smaller terms

$$= \int_0^L \left[1 + \frac{8h^2}{L^4} (L-2x)^2 \right] dx$$

$$= L + \frac{8h^2}{L^4} \left[L^2 \times L - \frac{4L \times L^2}{2} + \frac{4L^3}{3} \right]$$

$$= L + \frac{8h^2}{L^4} \left[\frac{1}{3} L^3 \right]$$

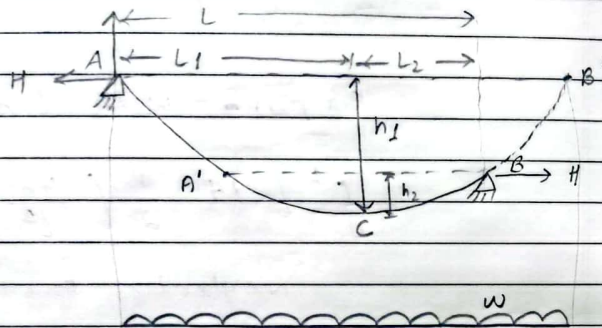
$$s = L + \frac{8h^2}{3L^2}$$

Say, $L = 100\text{m}$
 $h = 10\text{m}$,

$$S = 100 + \frac{8 \times 10^2}{3 \times 100}$$

$$[S = \underline{102.667\text{m}}]$$

* supports at different level:-



Cable A-C-B' (same level cable) Fig. (i)
 total span = $2L_1$

$$H = \frac{w(2L_1)^2}{8h_1}$$

Cable A'-C-B

total span = $2L_2$

$$H = \frac{w(2L_2)^2}{8h_2}$$

But, H is same, Equating.

$$\frac{w(2L_1)^2}{8h_1} = \frac{w(2L_2)^2}{8h_2}$$

$$\left[\frac{L_1}{L_2} = \sqrt{\frac{h_1}{h_2}} \right] \underline{u}$$

$$\text{or } \frac{L_1}{L_2} + 1 = \frac{\sqrt{h_1}}{\sqrt{h_2}} + 1$$

$$\text{on } \frac{L_1 + L_2}{L_2} = \frac{\sqrt{h_1} + \sqrt{h_2}}{\sqrt{h_2}}$$

$$\text{on } \left[L_2 = \frac{L \sqrt{h_2}}{\sqrt{h_1} + \sqrt{h_2}} \right] \underline{u}$$

Similarly,

$$\left[L_1 = \frac{L \sqrt{h_1}}{\sqrt{h_1} + \sqrt{h_2}} \right] \underline{u}$$

length of cable

$$\underline{ACB'} = 2L_1 + \frac{8}{3} \frac{h_1^2}{2L_1}$$

$$\underline{A'CB} = 2L_2 + \frac{8}{3} \frac{h_2^2}{2L_2}$$

$$\text{But, } A-C-B = \frac{ACB'}{2} + \frac{A'CB}{2}$$

$$\underline{u} \left[\text{length of cable, } S = (L_1 + L_2) + \frac{2}{3} \left(\frac{h_1^2}{L_1} + \frac{h_2^2}{L_2} \right) \right] \underline{u}$$

$$\left[S = L + \frac{2}{3} \left(\frac{h_1^2}{L_1} + \frac{h_2^2}{L_2} \right) \right] \underline{u}$$

$$* H = \frac{w(2L_1)^2}{8h_1} = \frac{w(2L_2)^2}{8h_2}$$

$$= w \left[2 \times \frac{L \sqrt{h_1}}{\sqrt{h_1} + \sqrt{h_2}} \right]^2 \frac{1}{8h_1}$$

$$\left[H = \frac{wL^2 h_1}{(\sqrt{h_1} + \sqrt{h_2})^2} \right] \underline{u} \text{ H for cable at diff. level.}$$

$$M_C = 0 \quad (\text{at } C)$$

$$\text{or, } V_A \times L_1 - w \times \frac{L_1 \times L_1}{2} - H \times h_1 = 0$$

$$\text{or, } V_A \times L_1 - \frac{w L_1^2}{2} - H \times h_1 = 0$$

$$\text{or, } V_A \times L_1 - \frac{w L_1^2}{2} - \frac{w L_1^2 \cdot h_1}{2 (\sqrt{h_1} + \sqrt{h_2})^2} = 0$$

$$\text{Substituting } \left[L_1 = \frac{L \sqrt{h_1}}{(\sqrt{h_1} + \sqrt{h_2})^2} \right]$$

$$L = \frac{L_1 (\sqrt{h_1} + \sqrt{h_2})^2}{\sqrt{h_1}}$$

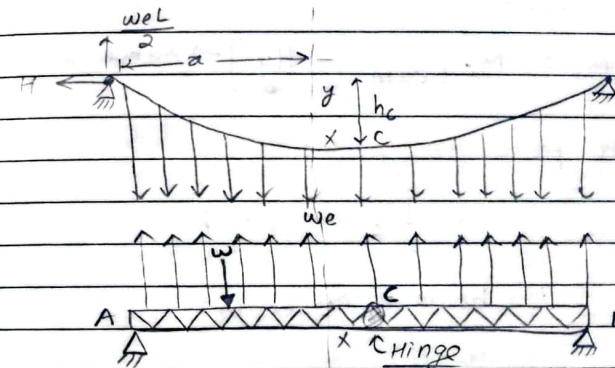
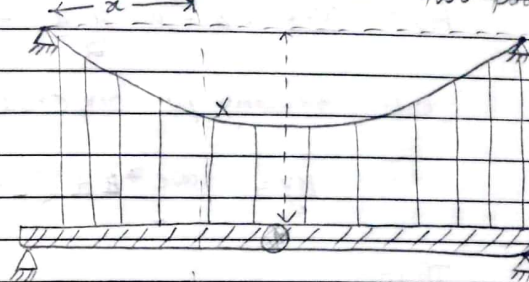
$$\text{Thus, } V_A \times L_1 - \frac{w L_1^2}{2} - \frac{w \cdot h_1 \times \frac{L_1^2 \times (\sqrt{h_1} + \sqrt{h_2})^2}{h_1}}{2 (\sqrt{h_1} + \sqrt{h_2})^2} = 0$$

$$\text{or, } V_A = \frac{w L_1}{2} \quad \text{same side fixed end length half}$$

$$\left\{ \begin{array}{l} T_{\text{max}} \rightarrow \text{at } A \\ T_{\text{max}} = \sqrt{H^2 + V_A^2} \end{array} \right\}$$

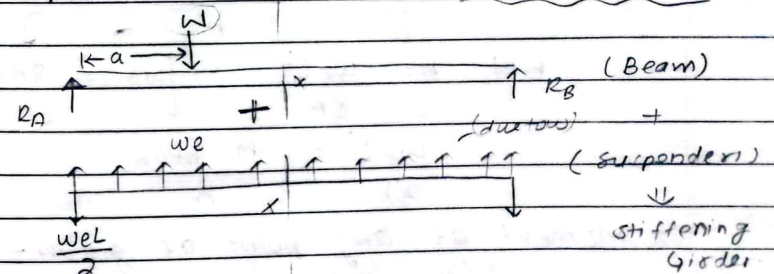
* Three hinged Stiffening Girders

Acc. to superposition principle seen in two parts.



we equivalent three hinged stiffening girder

we = equivalent UDL in cable due to live load (W)



$$\text{At any pt. } x, \quad M_x = R_A \cdot x - W(x-a) - \frac{w \cdot L \cdot x}{2} + \frac{w \cdot x \cdot x}{2}$$

$$M_x = M_{x, \text{Beam}} - \frac{w \cdot a}{2} (L-x)$$

In cable,

$$M_x = \frac{w_e L}{2} \cdot x - \frac{w_e x^2}{2} - H y.$$

But, at any pt. on cable, $M_x = 0$

$$\Rightarrow [H y = \frac{w_e x}{L} (L - x)]$$

Thus,

$$[M_x = M_{x, \text{beam}} - H y.] \rightarrow \text{same as three hinged arch.}$$

At pt. C in girder,

$$x = L/2, \quad y = h$$

$$M_{x, \text{beam}} = M_{c, \text{beam}}$$

$$M_c = 0 \quad (\text{Hinge})$$

$$\text{or, } 0 = M_{c, \text{beam}} - H \cdot h$$

considering girder as simply beam

$$\text{or, } [H = \frac{M_{c, \text{beam}}}{h}] \quad \text{we find}$$

$$\text{But, } H = \frac{w_e L^2}{8h} \Rightarrow [w_e = \frac{8Hh}{L^2}]$$

$$\frac{w_e L^2}{8h} = \frac{M_{c, \text{beam}}}{h}$$

* Moment at any point of girder -

$$[M_x = M_{x, \text{beam}} - H y]$$

Shear force,

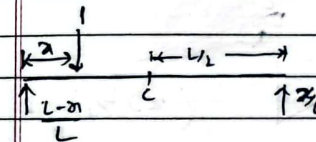
$$V_x = V_{x, \text{beam}} - \frac{w_e L}{2} + w_e x$$

$$= V_{x, \text{beam}} - \frac{w_e (L - 2x)}{2}$$

$$y = \frac{4h x (L - x)}{L^2}$$

$$\frac{dy}{dx} = \frac{4h (L - 2x)}{L^2} = \tan \theta$$

* ILD



$$H = \frac{M_{c, \text{beam}}}{h}$$

$$= \frac{x}{L} \cdot \frac{L}{2}$$

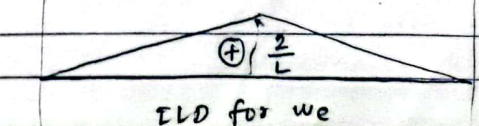
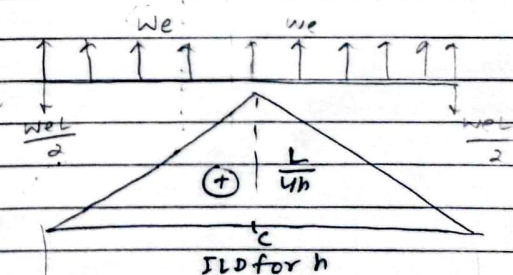
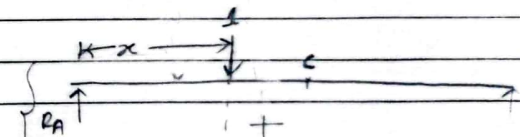
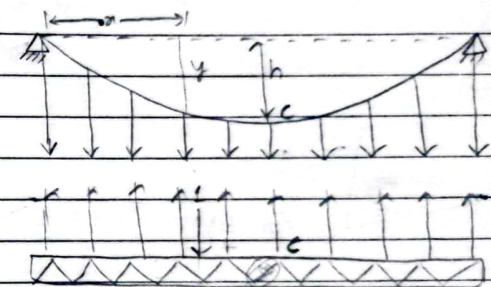
$$[H = \frac{x}{2h}]$$

for w_e ,

$$w_e = \frac{8Hh}{L^2}$$

$$= \frac{8 \times \frac{x}{2h} \times h}{L^2}$$

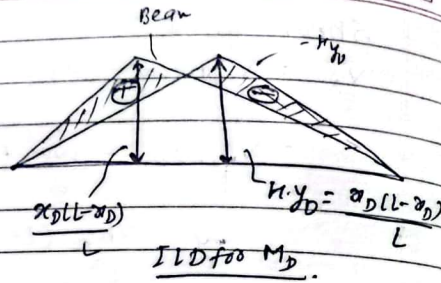
$$[w_e = \frac{4x}{L^2}]$$



For moment,

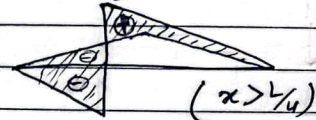
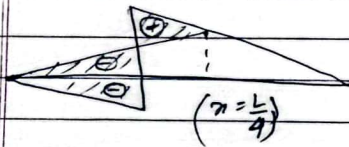
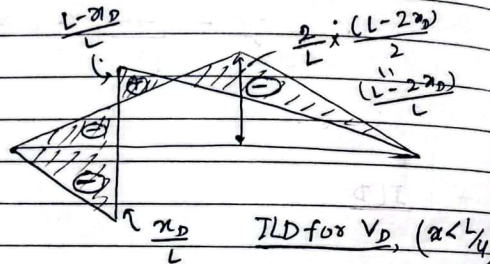
$$M_x = M_{x, \text{beam}} - Hy$$

$$M_D = M_{D, \text{beam}} - Hy_D$$

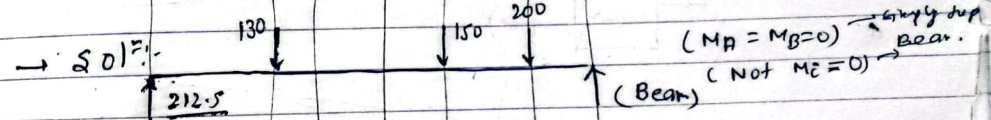
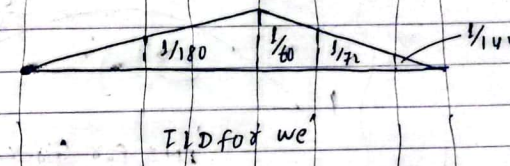
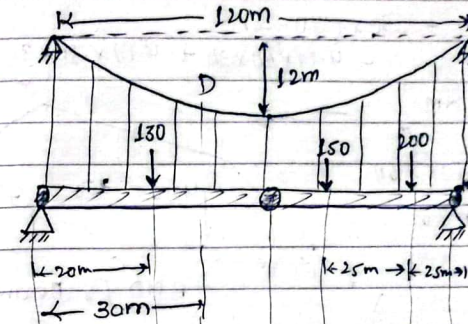


For shear force,

$$V_D = V_{D, \text{beam}} - \frac{w}{2}(L - 2x_D)$$



* Determine B.M. and S.F. at 30m from left support of girder. Max^e tension. Draw B.M.D, S.F.D,



$$= \frac{4.19 \times 120}{2} = \frac{w_e L}{2}$$

$$w_e = 130 \times \frac{1}{180} + \frac{1}{60} \times 0 + \frac{1}{72} \times 150 + \frac{1}{144} \times 200$$

$$= 4.194 \text{ kN/m}$$

[If self wt = 10 kN/m

then $w_e = 14.194 \text{ kN/m}$

self wt. for → only for H & V
direct transfer

girder
+
BM
(not due to self wt.)

$$\left[\sinh^{-1} \phi = \frac{w}{H} x \right] w$$

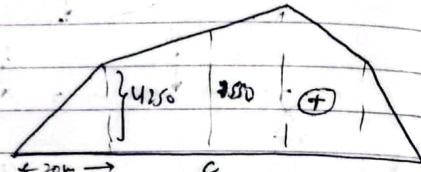
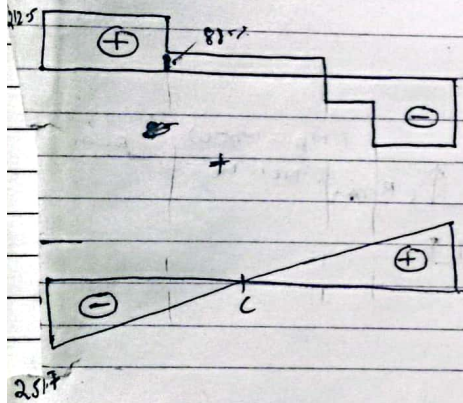
$$M_{30} = 212.5 \times 30 - 130 \times (30 - 60) - 4.19 \times 60 \times 30 + 4.19 \times 30 \times \frac{30}{2}$$

$$= -581.5 \text{ kNm}$$

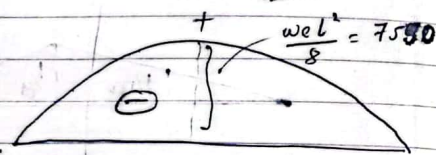
$$V_{30} = 212.5 - 130 - 4.19 \times 60 + 4.19 \times 30$$

$$= -43.2 \text{ kN}$$

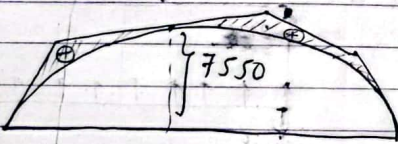
SFD for Beam



BMD for Beam

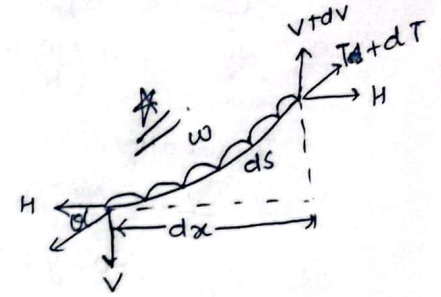
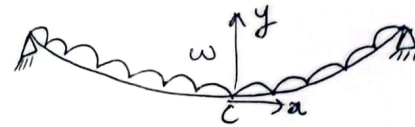


BMD for Girder



BMD for

Cable under self-weight (catenary)



→ shape is catenary (Electric cable)

→ Derivation of equation:

Let the wt. of cable per unit horizontal length = $\frac{w ds}{dx}$

$$H = T \cos \theta$$

$$V = T \sin \theta$$

$$\frac{V}{H} = \tan \theta \quad \text{--- (i)}$$

$$V = H \tan \theta = H \frac{dy}{dx} \quad \text{--- (ii)}$$

$$\sum F_y = 0$$

$$\text{or, } V + \left(\frac{w ds}{dx} \right) dx = V + dV$$

$$\text{or, } \frac{w ds}{dx} = \frac{dV}{dx} = \frac{d}{dx} \left(H \frac{dy}{dx} \right)$$

$$\text{or, } \frac{w ds}{dx} = H \frac{d^2 y}{dx^2}$$

$$\text{or, } \frac{d^2 y}{dx^2} = \frac{w}{H} \sqrt{1 + \left(\frac{dy}{dx} \right)^2}$$

$$\text{Let, } \frac{dy}{dx} = \phi$$

$$\left[\frac{d^2 y}{dx^2} = \frac{d\phi}{dx} \right]$$

Thus,

$$\frac{d\phi}{dx} = \frac{w}{H} \sqrt{1 + \phi^2}$$

$$\frac{w}{H} dx = \frac{d\phi}{\sqrt{1 + \phi^2}}$$

Integrating both side

$$\frac{w}{H} \int dx = \int \frac{d\phi}{\sqrt{1 + \phi^2}}$$

$$\text{or, } \frac{w}{H} x + c' = \sinh^{-1} \phi$$

$$\text{At } c, \quad x=0, y=0, \frac{dy}{dx} = \phi = 0$$

$$\therefore c' = 0$$

Thus,

$$\left[\sinh^{-1} \phi = \frac{w}{H} x \right]$$

$$\text{or, } \phi = \sinh \frac{w x}{H}$$

$$\text{or, } \frac{dy}{dx} = \sinh \left(\frac{w}{H} x \right)$$

Integrating,

$$y = \frac{H}{w} \cosh \left(\frac{w x}{H} \right) + C_2$$

$$\text{at, } C, \quad x=0, y=0$$

$$\therefore C_2 = -H/w$$

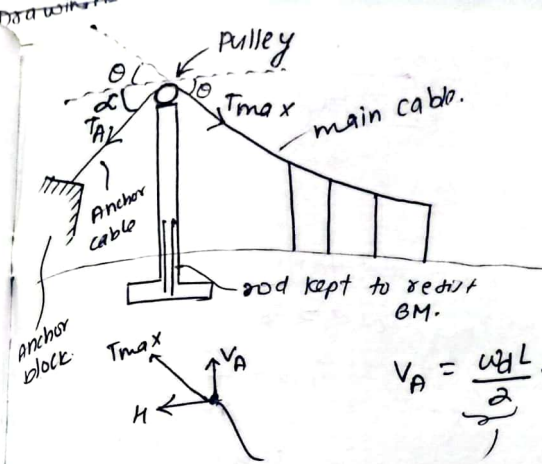
Equation of cable is,

$$\left\{ y = \frac{H}{w} \cosh \frac{w x}{H} - \frac{H}{w} = \frac{H}{w} \left[\cosh \left(\frac{w x}{H} \right) - 1 \right] \right\} \quad \leftarrow \text{Hyperbolic in nature.}$$

length of cable,

$$\begin{aligned} S &= \int ds = \int \frac{ds}{dx} \cdot dx \\ &= \int_0^x \sqrt{1 + \left(\frac{dy}{dx} \right)^2} dx \\ &= \int_0^x \sqrt{1 + \sinh^2 \frac{w x}{H}} dx \\ &= \int_0^x \cosh \frac{w x}{H} dx \end{aligned}$$

$$\left[S = \frac{H}{w} \sinh \frac{w x}{H} \right] \quad \leftarrow$$



$$V_A = \frac{w_d L}{2} + \frac{w_e L}{2}$$

$w_d \rightarrow$ udl due to dead load
 $w_e \rightarrow$ udl due to live load.

α
angle made by
Anchor cable
with horizontal
(can vary)

$$w_e = \frac{8 M_{c, beam}}{L^2}$$

$$H = \frac{w_d L^2}{8h} + \frac{w_e L^2}{8h}$$

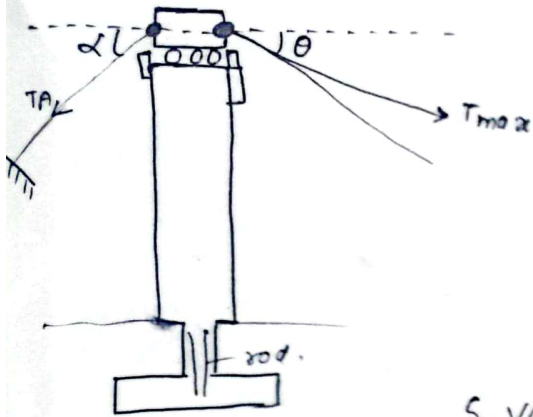
$$\text{For pulley, } T_{Max} = T_A = \sqrt{H^2 + V_A^2}$$

$$\left\{ \begin{array}{l} \text{Horizontal force on top of tower,} \\ H' = T_{Max} (\cos \alpha - \cos \theta) \end{array} \right\}$$

$\rightarrow$ develops moment on bottom of tower.

$$\sim \left[\text{Moment at bottom of tower} = \text{Horizontal force (H')} \times \text{height of tower} \right]$$

$$\sim \left[\text{Vertical force on tower (V')} = T_{Max} (\sin \theta + \sin \alpha) \right]$$



Horizontal force,

$$T_{max} \cos \theta = T_A \cos \alpha$$

$$[T_A = T_{max} \frac{\cos \theta}{\cos \alpha}]$$

$$\left\{ \begin{array}{l} \text{Vertical force in tower} \\ = T_{max} \sin \theta + T_A \sin \alpha \end{array} \right\}$$

Section modulus (Z) = $\frac{\text{Moment of inertia of section about neutral axis (I_{NA})}}{\text{Distance of outermost layer from neutral axis (y_{max})}}$

$$\frac{M}{I} = \frac{\sigma_{max}}{y_{max}}$$

$$M = \sigma_{max} \times \frac{I}{y_{max}} = \sigma_{max} \times Z$$

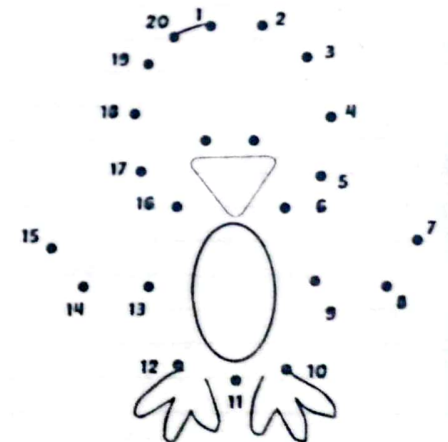
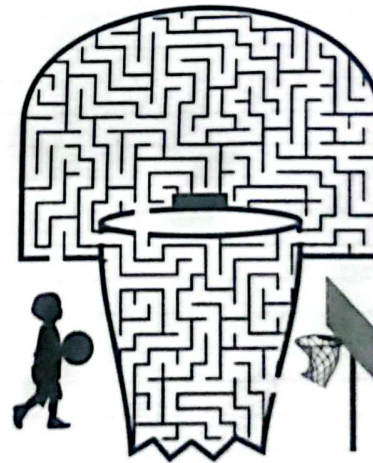
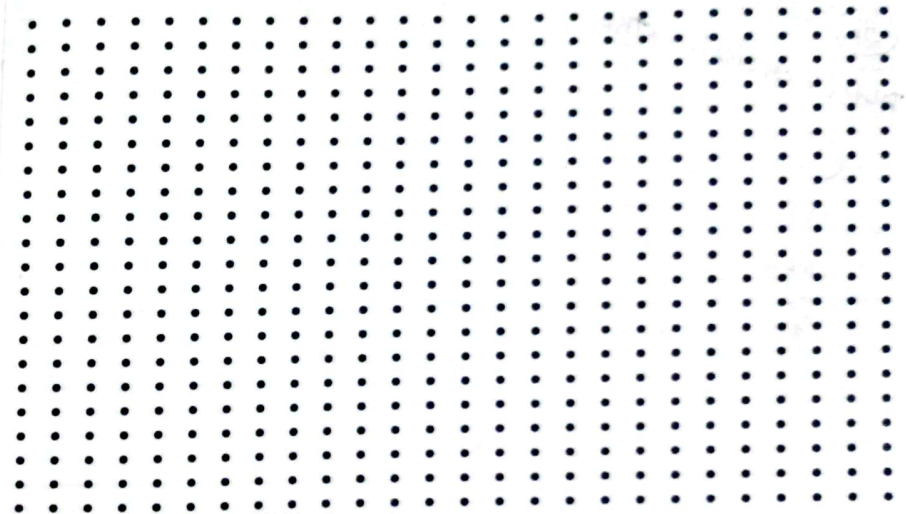
Let's play BRAINY DOTS & BOXES

How To Play?

Each player takes it turns to draw a line that joins two dots, either horizontally or vertically, the dots must be next to each other (no skipping over any dots).

when a player completes a whole box, they write their initial in the box.

when the whole grid is filled with boxes, the game is over and the person who has the most initials wins.



Never Stop Learning.....